Estimation of Electrical Conductivity and Magnetic Permeability of Metals from Surface Voltage Measurements

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Abstract: We consider a half-space conductor where the conductivity varies only in depth (the negative z direction). Denote by $\sigma(z)$ and $\mu(z)$ its conductivity and magnetic permeability, respectively. To determine the conductivity and the magnetic permeability, an alternating current is sent into the conductor and the electric potential is measured on the conductor's surface. Denote by ω the angular frequency of the injected current. The electric and magnetic fields in the conductor are controlled by the Maxwell's equations from which we obtain the following equation for the current density vector $\mathbf{J}$:

$$\nabla \times \nabla \times [\mathbf{J}(\mathbf{r})/\sigma(z)] - \imath\omega\mu(z)\mathbf{J}(\mathbf{r}) - \frac{\mu(z)'}{\mu(z)}\mathbf{a}_z \times \nabla \times \left[\frac{\mathbf{J}(\mathbf{r})}{\sigma(z)}\right] = 0, \; z < 0, \quad (3)$$

where $\mathbf{a}_z$ is the unit vector in the z-direction and $\mathbf{r} = (\rho, z)$. If the current at the surface enters the conductor only at an injection point located at the origin, (3) is coupled with the following boundary condition

$$J_z(\rho, 0) = \frac{I}{2\pi\rho}\delta(\rho), \quad (4)$$

where J_z is the z-component of the current density vector $\mathbf{J}$, I is the current intensity, and δ is the Diract delta function.

In this talk, we present a new method for determining the depth-varying coefficients $\sigma(z)$ and $\mu(z)$ from measurements of potential drops between two points on the conductor's surface ($z = 0$) at multiple frequencies, $\omega_1 < \omega_2 < \cdots < \omega_n$. The method consists of two steps: in the first step, using the Hankel transform $F(z, \kappa) = \int_0^\infty J_z(\rho, z) J_0(\kappa\rho) d\rho$, where J_0 is the Bessel function of first kind of zero order, we obtain from (3)–(4) a one-parameter set of boundary value problems for F. Then the potential drop data can be represented as a weighted integral of F over $(0, \infty)$.

Assuming that the variation of the coefficients are small, we linearize the inverse problem to obtain a system of linear integral equations. This system is then discretized and solved numerically.

The performance of the proposed method is illustrated with numerical tests with both simulated and real experimental data.

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