

# Asymptotic-preserving and Positivity-preserving Implicit-Explicit Schemes for the Stiff BGK Equation

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**Abstract:** We develop a family of second-order implicit-explicit (IMEX) schemes for the stiff BGK kinetic equation. The method is asymptotic-preserving (can capture the Euler limit without numerically resolving the small Knudsen number) as well as positivity-preserving — a feature that is not possessed by any of the existing second or high order IMEX schemes. The method is based on the usual IMEX Runge-Kutta framework plus a key correction step utilizing the special structure of the BGK operator. Formal analysis is presented to demonstrate the property of the method and is supported by various numerical results. Moreover, we show that the method satisfies an entropy-decay property when coupled with suitable spatial discretizations. Additionally, we discuss the generalization of the method to some hyperbolic relaxation system and provide a strategy to extend the method to third order.

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