The Insensitivity to Initial Condition of a New Stable Inverse

X. Ji[1] **and** **R. W. Longman**[2]

Abstract: Many applications of control theory aim to achieve zero tracking error. This is an inverse problem, given the desired output, compute the command to produce it. One would normally use a digital control for such an objective and then the input to the plant transfer function comes through a zero order hold, i.e. the input is held constant until an updated input arrives. In this case, the plant differential can be converted to a difference equation with the same solution at sample times. The inverse problems become one of substituting the desired output into the equations, and solving for the input. The conversion to a difference equation model introduces the input signal at different time steps, which results in asking to solve a nonhomogeneous difference equation for the desired control input. For nearly all plant systems governed by differential equation of third order or more, this difference equation which must be solved for the inverse problem is unstable, resulting in an unstable command input.

In order to avoid the exponential growth of the control action, we could ask that the initial conditions be such that the coefficient of the unstable solution of the homogeneous equation for the input be zero since it multiplies an exponentially growing term. This clearly appears to be very sensitive to getting the initial condition precisely correct. Recent developments by the authors have shown that not asking for zero error following the desired trajectory for the first time step (or several steps) allows a pseudo-inverse solution to produce stable control actions. In addition, we have used various Iterative Learning Control schemes that aim for zero tracking error, and these pick the initial condition based on the initial run made to start the learning process. One is free to use any command you want in the initial run. However, the result is that we never observe any sensitivity when producing inverse models.

The objective of this paper is to investigate why examining the problem from one point of view suggests that only very precise initial conditions can produce a stable inverse, while learning control experience always results in a stable inverse with no effort to fine tune the initial condition.

[1,2] Department of Mechanical Engineerin
Columbia University
MC4703, 500 West 120th Street, New York, NY 10027, USA
{*xj2145, rwl4*}*@columbia.ed*