Efficient Derivative Evaluation for Rigid-Body Dynamics

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Abstract: Mathematical models are key for simulation, optimization and control of the motions of technical, robotic and bio-mechanical systems. Usually, they are described as a system of rigid-bodies connect via joints which enable the use of highly efficient recursive algorithms to compute the required dynamic quantities of the multi-body system. More advanced models include spring-damper systems in addition to joint actuation and are subject to both kinematic and loop constraints. The latter employs tailored linear algebra to solve the respective descriptor form of the equation of motion, which allows an efficient evaluation of the dynamics of the system. To solve the forward problem, i.e. simulating an initial value problem for given control trajectories forward in time, having an efficient evaluation of the dynamics at hand is sufficient. However, state of the art methods for sensitivity analysis, model-based optimization or optimal control employ nonlinear programming algorithms which also rely heavily on the derivatives of the system w.r.t its inputs. Missing derivative information is often approximated by means of numerical differentiation, i.e. finite difference computations. However, nonlinear programming methods significantly benefit from better gradient information to handle the nonlinearity of the system dynamics as well as the ill-conditioning of the problem formulations, especially when the motions are kinematically constrained. In this talk, we show how the principle of automatic differentiation can be applied to propagate sensitivities through common recursive algorithms as well as the linear algebra required for kinematic and loop constraints. Following this, the approach is realized on algorithms implemented within the open-source software package RBDL. The proposed approach is thoroughly tested against its numerical differentiation counterpart on benchmark examples, a set multi-pendulums with branches, kinematic and loop constraints, two cart-pendulum examples and a whole-body model of a humanoid. Finally, we present results of optimal control problems consisting of time and energy efficient swing up of the cart-pendulums as well as the energy efficient walking motion of the humanoid model.

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