Inverse Problems of the Calculus of Variations for Systems of Discrete Equations of High Order

G. A. Kurina[1] **and V. G. Zadorozhniy**[2]

Abstract: The inverse problem of the discrete calculus of variations consists in the following. For a given system of discrete equations we want to know whether there is a functional for which the given system coincides with the Euler-Lagrange equation for this functional. If such a functional exists, then we need to find it. To our knowledge, inverse problems of the discrete calculus of variations were studied only in [1] for systems of discrete equations of the second and fourth orders.

In this talk we will present conditions ensuring the solvability of the inverse problem of the calculus of variations for the system

$$\varphi_k(x_{k-r}, x_{k-r+1}, \dots, x_{k+r}) = 0,\ k = 0, ..., N,$$

where $N > r$ and φ_k are continuously differentiable functions. If an variable x has a negative index or index is greater than N then this variable in the equation is absent. The case, where some boundary conditions are specified, is also studied. Explicit expressions of the functional of the form $\sum_{k=0}^{N-r} V_k(x_k, x_{k+1}, \dots, x_{k+r})$, for which the given system of discrete non-linear equations coincides with the Euler-Lagrange equation for this functional, are given.

Note that the inverse problems of the calculus of variations for continuous time are solved only for differential equations the order of which is not greater than six.

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References

[1] G. Kurina and V. Zadorozhniy. Inverse problems of the calculus of variations for discrete-time systems. Pure and Applied Functional Analysis . 2016. Vol. 1. No. 4, pp. 573-582.

[1] Federal Research Center "Computer Science and Control" of Russian Academy of Sciences
Vavilova ul. 44/2, 119333 Moscow, Russia;
Faculty of Mathematics, Voronezh State University
Universitetskaya pl. 1, 394018 Voronezh, Russia;
Institute of Law and Economics
Leninskii prospekt 119-A, 394000 Voronezh, Russia
kurina@math.vsu.ru

[2] Faculty of Applied Mathematics, Informatics and Mechanics, Voronezh State University
Universitetskaya pl. 1, 394018 Voronezh, Russia
zadorozhny@amm.vsu.ru