Radiation Conditions for Periodic Potentials

N. T. Ngoc[1] **and I. Witt**[2]

Abstract: The stationary Schrödinger operator $H = -\Delta + V(x)$ on $L^2(\mathbb{R}^d), d \geq 2$ with a periodic potential V has been intensively studied for many years by both mathematicians and physicists. It plays an important role in solid state physics and has applications to photonic crystal, metameterial, carbon nanostructure, and topological insulator. The main tool for periodic operators, Floquet-Bloch-Gelfand transform, reduces the study of a periodic operator on whole space $\mathbb{R}^d$ to the study of this operator on a bounded domain with various boundary conditions. Perturbation theory exhibits the band gap structure of the spectrum of the operator H. The main purpose of this talk is to study radiation conditions for the corresponding periodic Schrödinger equation

$$-\Delta u(x) + V(x)u(x) - \lambda u(x) = f(x), \tag{10}$$

where V is a smooth real-valued function on $\mathbb{R}^d$, periodic with respect to the integer lattice $\mathbb{Z}^d$, $f \in C_c^\infty(\mathbb{R}^d)$ and λ is the spectral parameter in the spectrum of H. Radiation conditions model wave propagation in periodic media when the frequency is in the spectrum. The radiation conditions are derived for energy level λ closed to the bottom of the spectrum. I will also discuss other cases with geometric conditions on the Fermi surface.

[1] Institute of Mathematics
University of Göttingen
Bunsenstrasse 3-5, D-37073 Gttingen, Germany
and
Department of Mathematics
School of Applied Mathematics and Informatics
Hanoi University of Science and Technology
No 1. Dai Co Viet, Hanoi, Vietnam
thaingoc273@gmail.com

[2] Institute of Mathematics
University of Göttingen
Bunsenstrasse 3-5, D-37073 Gttingen, Germany
iwitt@mathematik.uni-goettingen.de