

# Asymptotic Behavior of Singularly Perturbed Control Systems

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**Abstract:** In this talk we are interested in asymptotic behavior of singularly perturbed control systems. More precisely, we consider the value function of finite horizon optimal control problem (Bolza form) associated with singularly perturbed control systems, and aim at characterizing its weak semilimits as viscosity sub- and supersolutions of a limiting Hamilton-Jacobi-Bellman equation (effective HJB equation). This PDE approach is extensively studied in a series of papers by Alvarez & Bardi in the periodic setting.

Our contribution is to extend the results of Alvarez & Bardi to the non-periodic case. The key idea is to replace the periodicity on the datum by coercivity on the running cost, and we only need the local version of bounded-time controllability. The remarkable novelty of our work is to use some basic tools of Aubry-Mather theory developed by Fathi & Siconolfi for convex, coercive Hamiltonian. We finally obtain some similar results as those of Alvarez & Bardi.

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