An Efficient Primal-Dual Algorithm
for Fair Combinatorial Optimization

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Abstract: We consider a general class of combinatorial optimization problems including among others allocation, multiple knapsack, matching or travelling salesman problems. The standard version of those problems is the maximum weight optimization problem where a sum of values is optimized. However, the sum is not a good aggregation function when the fairness of the distribution of those values (corresponding for example to different agents' utilities or criteria) is important. In this paper, using the generalized Gini index (GGI), a well-known inequality measure, instead of the sum to model fairness, we formulate a new general problem, that we call fair combinatorial optimization. Although GGI is a non-linear aggregating function, a $0, 1$-linear program (IP) can be formulated for finding a GGI-optimal solution by exploiting a linearization of GGI proposed by Ogryczak and Sliwinski (European Journal of Operational Research, vol. 48, page 80-91, 2003). However, the time spent by commercial solvers (e.g., CPLEX, Gurobi...) for solving (IP) increases very quickly with instances' size and can reach hours even for relatively small-sized ones. As a faster alternative, we propose a heuristic for solving (IP) based on a primal-dual approach using Lagrangian decomposition. We demonstrate the efficiency of our method by evaluating it against the exact solution of (IP) by CPLEX on several fair optimization problems related to matching. The numerical results show that our method outputs in a very short time efficient solutions giving lower bounds that CPLEX may take several orders of magnitude longer to obtain. Moreover, for instances for which we know the optimal value, these solutions are quasi-optimal with optimality gap less than 0.3%.

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