Approach of an Epidemic Model by the Usage of a Dual Based Pseudospectral Method for Infinite Horizon Optimal Control Problems

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Abstract: We consider a class of infinite horizon optimal control problems with vector-valued states and controls involving the Lebesgue integral in the objective and a dynamics linear with respect to the control.

This special class of problems arises in epidemic models, in the theory of economic growth and in processes where the time T is an exponentially distributed random variable.

We consider an epidemic model in the form of a SEIR-model. The aim is to find an optimal vaccination strategy that prevents the spreading of the epidemic. Thereby we have to solve an optimal control problem with a quadratic objective and a nonlinear dynamic with respect to the states.

The problem is formulated as an optimization problem in Hilbert Spaces. The remarkable on this statement is the choice of Weighted Sobolev- and Weighted Lebesgue spaces as state and control spaces respectively.

These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a Hilbert space.

For the class of problems proposed, we are able to derive a related dual program in form of a variational problem in Hilbert Spaces by using the integrated Hamiltonian defect and formulate a Pseudo Maximum Principle for these problems.

Based on these principles we use an indirect pseudospectral method introduced in [1], to solve the problem numerically.

References

[1] S. Pickenhain, A. Burtchen, K. Kolo, and V. Lykina, An indirect pseudospectral method for the solution of linear-quadratic optimal control problems with infinite horizon, Optimization, Vol. 65, pp. 609–633, 2016.

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