How to Solve NP-hard Euclidean Geometry Problems: From Disk Packing Problem Point of View

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Abstract: Many hard problems such as disk packing is inherently geometric. This talk presents how to solve the disk packing problem which is a well-known classical NP-hard problem. Given the radii of disks, disk packing is to find the smallest container which encloses the disks without any overlap among disks and is also to find the corresponding coordinates of disk centers. There are some options for disk packing: The container is a circle, a rectangular, or simple polygon; The disk size is equal or unequal. We discuss how to pack circular disks within circular container where the disk size is not equal to each other. The disk packing can be formulated a non-linear program with a linear objective function and quadratic constraints. It turns out that the vacancy information around disks could be exploited to get good feasible solutions from the previous studies. We develop an efficient heuristic to solve the disk packing using the Voronoi diagram (VD) of disks. The VD of disks is the tessellation of space such that each cell of the tessellation consists of the locations which are closer to the corresponding disk than any others. Once we have VD, we could efficiently answer to various spatial queries such as the vacancy around disks. It turns out that the proposed algorithm efficiently and robustly computes the improved solution compared to the previous studies. The efficiency and robustness of the algorithm stem from those of VD.

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