Ba Vî, Hà Nội 19-21/4/2018

## TÓM TẮT BÁO CÁO

# Subdifferential Stability Analysis for Convex Optimization Problems via Multiplier Sets 

D. T. V. An ${ }^{1}$ and N. D. Yen ${ }^{2}$


#### Abstract

The present paper discusses differential stability of convex programming problems in Hausdorff locally convex topological vector spaces. Among other things, we obtain formulas for computing or estimating the subdifferential and the singular subdifferential of the optimal value function via suitable multiplier sets. Optimality conditions for convex optimization problems under inclusion constraints and for convex optimization problems under geometrical and functional constraints will be formulated too. But our main aim is to clarify the connection between the subdifferentials of the optimal value function and certain multiplier sets. Namely, by using some results from [1], we derive an upper estimate for the subdifferentials via the Lagrange multiplier sets and give an example to show that the upper estimate can be strict. Then, by defining a satisfactory multiplier set, we obtain formulas for exact computing the subdifferential and the singular subdifferential of the optimal value function.


## References

[1] D. T. V. An and N. D. Yen. Differential stability of convex optimization problems under inclusion constraints. Appl. Anal. 94, 108-128 (2015).

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# Well-posedness for Set Optimization Problems 

L. Q. Anh ${ }^{1}$, D. T. M. $\operatorname{Van}^{2}$, and P. T. Vui ${ }^{3}$


#### Abstract

In this report we consider set optimization problems involving set order relations and introduce many kinds of well-posedness for such problems. Then, we study sufficient and necessary conditions of these concepts of well-posedness for the reference problems. Moreover, by using Kuratowski measure of noncompactness, we present characterizations for the mentioned concepts. Relations of qualitative properties for solutions to set optimizations are also investigated.


[^0]
# On the Stability of Approximate Solutions to Set Valued Equilibrium Problems 

L. Q. Anh ${ }^{1}$, P. T. Duoc ${ }^{2}$, and T. N. Tam ${ }^{3}$


#### Abstract

We consider set valued equilibrium problems in locally convex Hausdorff topological vector spaces. By using linear scalarization technique for set, we study conditions for the stability of solutions to such problems. Some applications to optimization problem and variational inequality are also discussed.


[^1]
# Study on Qualitative Properties of Solutions to Split Equilibrium Problems 

L. Q. Anh ${ }^{1}$, N. P. Duc ${ }^{2}$, and T. Q. Duy ${ }^{3}$


#### Abstract

In this talk, we consider split equilibrium problems in Banach spaces. We study conditions for various kinds of qualitative properties of the solutions to such problems, including upper (lower) semicontinuity, closedness, well-posedness. Many examples are provided to ensure the essentialness of the imposed assumptions. As an application, we discuss the special case of split variational inequality.


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# Gap Function and Stability of Solution Mapping to Parametric Vector Quasi-Equilibrium Problems 

L. Q. Anh ${ }^{1}$, T. Q. Duy ${ }^{2}$, and D. V. Hien ${ }^{3}$


#### Abstract

In this paper we consider parametric vector quasi-equilibrium problems in normed spaces. Using nonlinear scalarization functions, we propose gap functions for such problems and investigate some their important properties. Then, by virtue of these gap functions, key assumptions are proposed and employed to study sufficient/necessary conditions of the Hausdorff continuity for solution mappings to the reference problems. Finally, as an application, we discuss a special case of traffic network problem.


[^2]
# On Hölder Continuity of Solution Map to the Parametric Optimal Control Problem 

L. Q. Anh ${ }^{1}$, T. N. Tam ${ }^{2}$, and V. T. Tai ${ }^{3}$


#### Abstract

In this report, we consider the parametric optimal control problem with linear state equation and control constraints. Sufficient conditions for Hölder calmness of solution maps are established. In addition, we also study the Hölder continuity of solution maps for the parametric equilibrium control problem.


[^3]
# The Tikhonov Regularization for Vector Equilibrium Problems 

L. Q. Anh ${ }^{1}$, T. Q. Duy ${ }^{2}$, L. D. Muu ${ }^{3}$, and T. V. Tri ${ }^{1}$


#### Abstract

We consider vector equilibrium problems in real Banach spaces and study their regularized problems. Based on cone continuity and generalized monotonicity of vectorvalued mappings, we provide conditions for the existence of solutions for such problems. We first show that every Tikhonov trajectory converges to a solution of the original problem. Then, we establish the equivalence between the problem solvability and the boundedness of any Tikhonov trajectory.


[^4]
# The Einstein Constraint Equations: Analytical Study and Numerical Simulations 

N. Q. Anh ${ }^{1}$


#### Abstract

It has been known that the famous Einstein equation in general relativity is essentially hyperbolic. The first step in studying solutions to Einstein's equations via Cauchy's problem is to understand the parameterization and the construction of sets of initial data which satisfy a so-called Einstein's constraint equations which is a highly under-determined system of partial differential equations.

Toward a fully understanding of solutions to the constraints, the conformal method has long been recognized to be an important tool for carrying out such analyses since the 1970s. Applying this method leads to a complete answer for the existence of solutions, however, were limited to the constant mean curvature (CMC) case through 1995, with analogous satisfying results for near-CMC cases beginning in 1996 and after. The first far-from-CMC result appeared in 2008 with various extensions and it was initially hoped that these new results would lead to a complete description for non-CMC cases. Unfortunately, the story has become much more complicated.


Part of this talk is to discuss numerical simulations to the study of the conformal method, in an attempt to give some insight into what the behavior of solutions is in far-from-CMC cases in various scenarios necessary for the direction of further study of the constraints.

Trong vật lý toán, người ta biết rằng phương trình Einstein trong thuyết tương đối rộng có tính chất sóng. Để nghiên cứu nghiệm của phương trình Einstein thông qua bài toán Cauchy, chúng ta cần tìm hiểu khi nào thì dữ liệu ban đầu được chọn là thích hợp. Bài toán xác định sự phù hợp của dữ liệu ban đầu dẫn đến các phương trình ràng buộc Einstein; đó là một hệ các phương trình vi phân đạo hàm riêng.

Từ những năm 1970 phương pháp bảo giác đã được công nhận là một công cụ quan trọng và hữu hiệu để thực hiện các nghiên cứu cho hệ phương trình ràng buộc Einstein. Một trong các kết quả quan trọng khi sử dụng phương pháp bảo giác là tính giải được của hệ phương trình ràng buộc trong trường hợp độ cong trung bình là hằng số (1995) và các kết quả tương tự cho các trường hợp độ cong trung bình gần hằng số (1996). Các kết quả về tính giải được của hệ phương trình rằng buộc với độ cong trung bình bất kỳ xuất hiện lần đầu trong năm 2008. Kết quả đột phá này cùng với các mở rộng khác nhau của chúng cho phép chúng ta hi vọng rằng câu hỏi về tính giải được của hệ phương trình ràng buộc trong trường hợp tổng quát sẽ nhanh chóng được giải quyết. Thật không may, câu chuyện trở nên phức tạp hơn rất nhiều.

Một phần của bài nói chuyện này nhằm mục đích thảo luận các mô phỏng số cho hệ phương trình ràng buộc Einstein trong một số trường hợp đặc biệt nhằm cung cấp một cái nhìn sâu hơn về dáng điệu nghiệm của các phương trình ràng buộc trong các trường hợp với độ cong trung bình bất kỳ. Các mô phỏng số này là cần thiết vì nó giúp định hướng cho việc nghiên cứu các phương trình ràng buộc trong trường hợp tổng quát.

[^5]
# A Subgradient Proximal-like Algorithm for Multivalued Variational Inequalities 

P. N. Anh ${ }^{1}$


#### Abstract

Let $C$ be a nonempty closed convex subset of a finite dimensional vector space $\mathcal{R}^{n}$, a multivalued mapping $F: \mathcal{R}^{n} \rightarrow 2^{\mathcal{R}^{n}}$ and a convex function $g: C \rightarrow(-\infty,+\infty]$. This paper considers a problem of the multivalued variational inequalities in a general form:


$$
\text { Find } x^{*} \in C \text { and } w^{*} \in F\left(x^{*}\right) \text { such that }\left\langle w^{*}, x-x^{*}\right\rangle+g(x)-g\left(x^{*}\right) \geq 0 \quad \forall x \in C \text {. }
$$

In this paper, we propose and analyze the convergence of a new subgradient proximal-like algorithm for solving monotone and Lipschitz continuous multivalued variational inequalities. The algorithm is a variant of the proximal-like method and the subgradient method. By choosing suitable parameter sequences of proximal steps and of subgradient stepsizes, we show the convergence of the subgradient proximal-like algorithm. Moreover, the establishing of convergence does not require the prior knowledge of Lipschitz continuous constant of cost operators.

[^6]
# A Farkas Lemma Approach to Calmness of Linear Inequality Systems 

M. J. Cánovas ${ }^{1}$, N. Dinh ${ }^{2}$, D. H. Long ${ }^{3}$, and J. Parra ${ }^{4}$


#### Abstract

We deal with the feasible set mapping of linear inequality systems under righthand side perturbations. From a version of Farkas lemma for difference of convex functions, we derive an operative relationship between calmness constants for this mapping at a nominal solution and associated neighborhoods where such constants work. Illustrative examples are provided with the aim of showing how this approach also allows us to compute the sharp Hoffman constant in specific situations.


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# Graphical Derivative of Subdifferential Mapping with Some Applications 

N. H. Chieu ${ }^{1}$


#### Abstract

Graphical derivative of a set-valued mapping at a point in its graph is the setvalued mapping whose graph is the (Bouligand-Severi) tangent/contingent cone to the graph of the given set-valued mapping at the point in question. This concept also known as the outer graphical derivative was introduced in 1981 by J.-P. Aubin who called it the contingent derivative. It is a powerful tool in variational analysis. In this talk, we will present some recent results on graphical derivative of subdifferential mapping with various applications. In particular, we will show that graphical derivative of subdifferential mappings can be used to examine tilt-stability for optimization problems and isolated calmness for variational systems. Furthermore, we will clarify relationships between the so-called positive definitenness of graphical derivative of subdifferential mappings and well-known second-order optimality conditions. Besides, we will show how to compute graphical derivative of subdifferential mappings.


This talk is based on joint works with L. V. Hien, T. T. A. Nghia, and H. A. Tuan.

[^7]
# Meshless Finite Difference Method for Elliptic Equations on Complicated 3D Domains 

O. Davydov ${ }^{1}$, N. M. Tuong ${ }^{2}$, and D. T. Oanh ${ }^{3}$


#### Abstract

Adaptive meshless finite difference method relying on radial basis function stencils developed in $[1,2]$ for elliptic equations in 2D has shown competitive performance on benchmark problems in comparison to the state-of-the art finite element techniques based on a posteriori error estimates. We discuss recent research aiming at extending this method to 3D, with emphasis on complicated domains, where the meshless nature of the method is a significant advantage. In particular, new algorithms for the selection of sets of influence will be presented.


## References

[1] O. Davydov and D. T. Oanh. Adaptive meshless centres and RBF stencils for Poisson equation. J. Comput. Phys. 230 (2011), 287-304.
[1] D. T. Oanh, O. Davydov, and H. X. Phu. Adaptive RBF-FD method for elliptic problems with point singularities in 2D. Appl. Math. Comput. 313 (2017), 474-497.

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# Convexity and Closedness in Stable Robust Duality 

N. Dinh ${ }^{1}$, M. A. Goberna ${ }^{2}$, M. A. Lopez ${ }^{3}$, and M. Volle ${ }^{4}$


#### Abstract

The paper deals with optimization problems with uncertain constraints and linear perturbations of the objective function, which are associated with given families of perturbation functions. More in detail, the paper provides characterizations of stable strong robust duality and stable robust duality under convexity and closedness assumptions. The paper also reviews the classical Fenchel duality of the sum of two functions by considering a suitable family of perturbation functions.


[^8]
# Ửng dụng điều khiển tối ưu vào tính toán Cơ học 

N. T. Đôn ${ }^{1}$

Tóm tắt: Ở nước ngoài, ứng dụng toán học nói chung, ứng dụng diều khiển tối ưu nói riêng vào tính toán Cơ học đã đem lại hiệu quả rất cao. Ở Việt Nam, nhu cầu ứng dụng điều khiển tối ưu còn rất cao, nhưng có nhiều trở ngại triển khai rất chậm, thấy rõ sự tụt hậu so với các nước phát triển. Báo cáo này trình bày về một số nhu cầu thực tế, bài toán cụ thể và một số thành công ban đầu trong lĩnh vực thủy động lực học nói riêng để thấy được sự cấp thiết tiếp tục phát triển các ứng dụng của điều khiển tối ưu vào Cơ học.

[^9]
# Contraction Mapping Method for the Equilibrium Problem over the Fixed Point Set 

T. N. Hai ${ }^{1}$


#### Abstract

In this paper, we introduce a new definition of Lipschitz-type continuity of a bifunction. Using this definition, we prove the contraction of the proximal mapping and apply it to the equilibrium problem over the fixed-point set of a nonexpansive mapping. We present a new algorithm for this problem. Under classical conditions, the convergence of the algorithm is proved.


[^10]
# New Inertial Algorithm for a Class of Equilibrium Problems 

D. V. Hieu ${ }^{1}$


#### Abstract

The article introduces a new algorithm for solving a class of equilibrium problems involving strongly pseudomonotone bifunctions with a Lipschitz-type condition. We describe how to incorporate the proximal-like regularized technique with inertial effects. The main novelty of the algorithm is that it can be done without previously knowing the information on the strongly pseudomonotone and Lipschitz-type constants of cost bifunction. A reasonable explain for this is that the algorithm uses a sequence of stepsizes which is diminishing and non-summable. Theorem of strong convergence is proved. In the case, when the information on the modulus of strong pseudomonotonicity and Lispchitz-type constant is known, the rate of linear convergence of the algorithm has been established. Several of experiments are performed to illustrate the numerical behavior of the algorithm and also compare it with other algorithms.


Keywords: Proximal-like method; Regularized method; Equilibrium problem; Strongly pseudomonotone bifunction; Lipschitz-type bifunction.

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# Commutative Algebra and Optimization 

L. T. Hoa ${ }^{1}$


#### Abstract

In this talk I will review some new results in Commutative Algebra in order to show connection between Commutative Algebra and (integral) linear programming. I will mainly focus on research carried out by Vietnamese mathematicians.


## References

[1] C. Bocci, S. Cooper, E. Guardo, B. Harbourne, M. Janssen, U. Nagel, A, Seceleanu, A. Van Tuyl, and T. Vu. The Waldschmidt constant for squarefree monomial ideals. J. Algebraic Combin. 44 (2016), no. 4, 875-904.
[2] E. Ehrhart. Sur un problème de géométrie diophantienne linéaire. I. Polyèdres et réseaux. (French) J. Reine Angew. Math. 226 (1967), 1-29.
[3] E. Ehrhart. Sur un problème de géométrie diophantienne linéaire. II. Systèmes diophantiens linéaires. (French) J. Reine Angew. Math. 227 (1967), 2-49.
[4] D. H. Giang and L. T. Hoa. On local cohomology of a tetrahedral curve. Acta Math. Vietnam. 35 (2010), 229-241.
[5] H. T. Ha, D. H. Nguyen, N. V. Trung, and T. N. Trung. Symbolic powers of sums of ideals. Preprint ArXiv:1702.01766.
[6] J. Herzog and T. Hibi. Monomial ideals. Graduate Texts in Mathematics, 260. Springer-Verlag London, Ltd., London, 2011. xvi+305 pp.
[7] L.T. Hoa. Stability of associated primes of monomial ideals. Vietnam J. Math. 34 (2006), no. 4, 473-487.
[8] L. T. Hoa, K. Kimura, N. Terai, and T. N. Trung. Stability of depths of symbolic powers of StanleyReisner ideals. J. Algebra. 473 (2017), 307-323.
[9] L. T. Hoa and T. N. Trung. Partial Castelnuovo-Mumford regularities of sums and intersections of powers of monomial ideals. Math. Proc. Cambridge Philos Soc. 149 (2010), 1-18.
[10] L. T. Hoa and T. N. Trung. Castelnuovo-Mumford regularity of symbolic powers of two-dimensional square-free monomial ideals. J. Commut. Algebra. 8 (2016), no. 1, 77-88.
[11] L. T. Hoa and T. N. Trung. Stability of Depth and Cohen-Macaulayness of Integral Closures of Powers of Monomial Ideals. Acta Math. Vietnam. DOI 10.1007/s40306-017-0225-0, Preprint ArXiv 1706.07603
[12] E. Miller and B. Sturmfels. Combinatorial commutative algebra. Graduate Texts in Mathematics, 227. Springer-Verlag, New York, 2005. xiv +417 pp.
[13] N. C. Minh and N. V. Trung. Cohen-Macaulayness of monomial ideals and symbolic powers of Stanley-Reisner ideals. Adv. Math. 226 (2011), no. 2, 1285-1306.
[14] R. Stanley. Combinatorics and commutative algebra. Second edition. Progress in Mathematics, 41. Birkhäuser Boston, Inc., Boston, MA, 1996. x+164pp.
[15] Y. Takayama. Combinatorial characterizations of generalized Cohen-Macaulay monomial ideals. Bull. Math. Soc. Sci. Math. Roumanie (N.S.). 48 (2005), 327-344.
[16] N. Terai and N. V. Trung. Cohen-Macaulayness of large powers of Stanley-Reisner ideals. Adv. Math. 229 (2012), no. 2, 711-730.
[17] T. N. Trung. Stability of depths of powers of edge ideals. J. Algebra. 452 (2016), 157-187.
[18] R. H. Villarreal. Monomial algebras. Second edition. Monographs and Research Notes in Mathematics. CRC Press, Boca Raton, FL, 2015. xviii+686 pp.

[^11]
# Combinations of Boolean Gröbner Bases and SAT Solvers 

N. T. Hung ${ }^{1}$ and N. T. Kien ${ }^{2}$


#### Abstract

We combine Gröbner basis with SAT Solver in different manners. Both SAT solvers and Gröbner basis techniques have their own strength and weakness. Combining them could fix their weakness.

The first combination is using Gröbner techniques to learn additional binary clauses for SAT solver from a selection of clauses. This combination is first proposed by Zengler and Küchlin. However, in our experiments, about 80 percent Gröbner basis computations give no new binary clauses. By selecting smaller and more compact input for Gröbner basis computations, we can significantly reduce the number of inefficient Gröbner basis computations, learn much more binary clauses. In addition, the new strategy can reduce the solving time of a SAT Solver in general, especially for large and hard problems.

The second combination is using all-solution SAT solver and interpolation to compute Boolean Gröbner bases of Boolean elimination ideals of a given ideal. Computing Boolean Gröbner basis of the given ideal is an inefficient method in case we want to eliminate most of the variables from a big system of Boolean polynomials. Therefore, we propose a more efficient approach to handle such cases. In this approach, the given ideal is translated to the CNF formula. Then an all-solution SAT Solver is used to find the projection of all solutions of the given ideal. Finally, an algorithm, e.g. Buchberger-Moeller Algorithm, is used to associate the reduced Gröbner basis to the projection. We also optimize the Buchberger-Moeller Algorithm for lexicographical ordering and compare it with Brickenstein's interpolation algorithm.


Finally, we combine Gröbner basis and abstraction techniques to the verification of some digital designs that contain complicated data paths. For a given design, we construct an abstract model. Then, we reformulate it as a system of polynomials in the ring $\mathbb{Z}_{2^{k}}\left[x_{1}, \ldots, x_{n}\right]$. The variables are ordered in a way such that the system has already been a Gröbner basis w.r.t lexicographical monomial ordering. Finally, the normal form is employed to prove the desired properties. To evaluate our approach, we verify the global property of a multiplier and a FIR filter using the computer algebra system Singular. The result shows that our approach is much faster than the commercial verification tool from Onespin on these benchmarks.

[^12]
# Proper Efficiency in Linear Fractional Vector Optimization 

N. T. T. Huong ${ }^{1}$, J.-C. Yao ${ }^{2}$, and N. D. Yen ${ }^{3}$


#### Abstract

This paper studies the properness in the sense of Geoffrion of the efficient solutions in linear fractional vector optimization. By examples, it will be shown that linear fractional vector optimization problems may have improperly efficient solutions. Sufficient conditions for an efficient solution of a linear fractional vector optimization to be a properly efficient solution are obtained. We distinguish two cases: a) problems with bounded constraint sets; b) problems with unbounded constraint sets.


[^13]
# Differentiability Properties of a Parametric Consumer Problem 

V. T. Huong ${ }^{1}$, J.-C. Yao ${ }^{2}$, and N. D. Yen ${ }^{3}$


#### Abstract

We study the budget map and the indirect utility function of a parametric consumer problem in a Banach space setting by some advanced tools from set-valued and variational analysis. The Lipschitz-likeness and differentiability properties of the budget map, as well as formulas for finding subdifferentials of the infimal nuisance function, which is obtained from the indirect utility function by changing its sign, are established. Our investigation is mainly based on the paper by Mordukhovich [J. Global Optim. 28 (2004), 347-362] on coderivative analysis of variational systems and the paper of Mordukhovich, Nam, and Yen [Math. Program. 116 (2009), 369-396] on subgradients of marginal functions. Economic meanings of the obtained subdifferential estimates are explained in details.


[^14]
# Inertial Manifolds and Feedback Control of Reaction-Diffusion Equations 

N. T. Huy ${ }^{1}$, B. X. Quang ${ }^{2}$, and D. D. Thuan ${ }^{3}$


#### Abstract

Since its first appearance in the work by C. Foias, G.R. Sell and R. Temam (1985) dealing with Navier-Stokes equations, the concept of inertial manifolds has played an important role in research for asymptotic behavior of solutions to evolution equations. An inertial manifold is a smooth finite-dimensional manifold of the phase space which is positively invariant and attracts exponentially all orbits. In this talk, using the inertial manifold theory, we will construct a finite-dimensional feedback controller for a one-dimensional semilinear reaction-diffusion equation.


[^15]
# The Inverse Connected $p$-Median Location Problem on Block Graphs 

K. T. Nguyen ${ }^{1}$ and N. T. Hung ${ }^{1}$


#### Abstract

A p-connected median on a graph is a set of $p$ vertices such that they are connected in the underlying graph and the total weighted distances from all other vertices to the given set is minimized. This paper addresses the problem of modifying vertex weights of a block graph at minimum cost so that a predetermined set of connected $p$ vertices become a connected $p$-median on the perturbed block graph. This problem is called the inverse connected $p$-median problem on block graphs. To solve this problem, we first construct an optimality criterion for a given set that is a connected $p$-median. This criterion can be observed as a generalization of the condition for a vertex to be a 1-median, i.e., $p=1$. Based on this criterion and the convexity of the cost function, we then develop a combinatorial algorithm that solves the problem in $O(n \log n)$ time, where $n$ is the number of vertices in the underlying block graph.


Keywords: Connected p-median, Inverse optimization, Convex, Knapsack, Block graphs

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# A Linear Time Algorithm for Balance Vertices on Trees 

T. T. Le ${ }^{1}$ and K. T. Nguyen ${ }^{1}$


#### Abstract

The concept of balance vertices was first investigated by Reid (1991). For the main result "the balance vertices of a tree consist of a single vertex or two adjacent vertices", Shan and Kang (2004), Reid and DePalma (2005) improved the length and technique of the proof. In this paper we further discuss the balance vertices on trees in a generalization context. We do not only provide a simple efficient proof for the relevant result but also develop a linear time algorithm to find the set of balance vertices on the underlying tree.


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# Piecewise Linear Vector Optimization Problems on Locally Convex Hausdorff Topological Vector Spaces 

N. N. Luan ${ }^{1}$


#### Abstract

Piecewise linear vector optimization problems in a locally convex Hausdorff topological vector spaces setting are considered in this paper. The efficient solution set of these problems are shown to be the unions of finitely many semi-closed generalized polyhedral convex sets. If, in addition, the problem is convex, then the efficient solution set and the weakly efficient solution set are the unions of finitely many generalized polyhedral convex sets and they are connected by line segments. Our results develop the preceding ones of Zheng and Yang [Sci. China Ser. A. 51, 1243-1256 (2008)], and Yang and Yen [J. Optim. Theory Appl. 147, 113-124 (2010)], which were established in a normed spaces setting.


[^16]
# Tính chính quy metric phi tuyến của ánh xạ đa trị trên một tập cố định 

H. V. Ngãi ${ }^{1}$, N. H. Trọn ${ }^{2}$, và Đ. N. Hân ${ }^{3}$

Tóm tắt: Trong bài báo này, chúng tôi thiết lập các đặc trưng của tính chính quy metric phi tuyến qua độ dốc địa phương, độ dốc toàn cục; đối đạo hàm và nghiên cứu tính ổn định của nó.

[^17]
# Geometric Properties for Level Sets of Quadratic Functions 

N. H. Quang ${ }^{1,2}$ and R. L. Sheu ${ }^{2}$


#### Abstract

This paper answers some mathematical problems related to quadratic functions. We prove two main theorems. First, The level set of a polynomial of degree 2 cannot separate the sublevel set of another polynomial of the same degree (see Theorem A). Secondly, the range of a polynomial of degree 2 on the sublevel set of another polynomial of the same degree is always connected (see Theorem B). As an application of Theorem A and Theorem $B$ to the generalized trust region subproblem (GTRS), we now are not only able to find the infimum (supremum) of (GTRS) but also able to find out the image of constraint set by objective function. Moreover the former theorems can be applied to provide an elegant and neat geometric proof for the S-lemma and the S-lemma with equality, whereas the latter one is an intermediate value theorem type of results.


Theorem A. Let $P_{d}$ be the set of all couples $(g, f)$ of polynomials of degree $d$ such that $\{g=$ $0\}$ separates $\{f \leq 0\}$ then $P_{d} \neq \emptyset$ if and only if $d \geq 3$. Moreover, let $f(x)=x^{T} A x+a^{t} x+a_{0}$, $g(x)=x^{T} B x+b^{t} x+b_{0}$ then $\{g=0\}$ separates $\{f \leq 0\}$ if and only if
(i) $f(x)$ is of the form $-x_{1}^{2}+\delta\left(x_{2}^{2}+\cdots+x_{m}^{2}\right)+\theta, \delta, \theta \in\{0,1\}$;
(ii) With the same basis and $\delta$ as in (i), $g(x)$ has the form $b_{1} x_{1}+\delta\left(b_{2} x_{2}+\cdots+b_{m} x_{m}\right)+$ $b_{0}, b_{1} \neq 0$;
(iii) $-\left(\delta \frac{b_{2}}{b_{1}} x_{2}+\cdots+\delta \frac{b_{m}}{b_{1}} x_{m}+\frac{b_{0}}{b_{1}}\right)^{2}+\delta\left(x_{2}^{2}+\cdots+x_{m}^{2}\right)+\theta \geq 0, \forall\left(x_{2}, \cdots, x_{n}\right)^{T} \in \mathbb{R}^{n-1}$.

In particular, if $B \neq 0,\{g=0\}$ cannot separate $\{f<0\}$.
Theorem B. Let $f, g$ are quadractic functions, then image set $g(\{f<0\})$ is disconnected if and only if there exists $\gamma \in g\left(\mathbb{R}^{n}\right)$ such that $\{g=\gamma\}$ separates $\{f<0\}$. Particularly, if $B \neq 0$, then $g(\{f<0\})$ is connected.

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# Semigroup of Operators Defined on the Set of Triangular Fuzzy Numbers and Its Application 

N. T. K. Son ${ }^{1}$ and H. V. Long ${ }^{2}$


#### Abstract

In this talk, we present the strongly continuous semigroups of fuzzy-valued operators defined on the space of triangular fuzzy numbers. New notions of fuzzy infinitesimal generators and fuzzy resolvent operators of fuzzy semigroups are established in sense of generalized Hukuhara difference. A Hille-Yosida like theorem in the framework of fuzzy metric space is investigated. We demonstrate the efficiency of theoretical results by studying the existence of mild solutions of fuzzy nonlinear fractional evolution equations.


[^18]
# Frequency Shifting Procedures for Solitons of the Nonlinear Schrödinger Equation 

H. T. Toan ${ }^{1,2}$ and N. M. Quan ${ }^{3}$


#### Abstract

In 1973, Hasegawa and Tappert at Bell Labs showed the existence of solitons in optical fiber, which can be described by the nonlinear Schrödinger (NLS) equation. This leads to the explosion of high-speed communication technologies based on solitons transmission in the last three decades. In this work, we propose the mathematical procedures of shifting frequency for solitons of the NLS equation in which other parameters of solitons are preserved. The procedures are based on simple transformations of the Fourier transform of the soliton pattern and on the shape-preserving property of solitons. We verify the theoretical frequency shifting procedures by numerical simulations of the NLS equation with using the split-step Fourier method.


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# A New Method for Designing Distributed Reduced-Order Functional Observers of Interconnected Time-delay Systems 

H. Trinh ${ }^{1}$ and D. C. Huong ${ }^{2}$


#### Abstract

This paper reports a new method for designing distributed reduced-order functional observers of a class of interconnected systems with time delays. The systems under consideration belong to a class of large-scale systems where each system is formed by a number of interconnected subsystems. Moreover, the interconnections and the states of the local subsystems are subject to heterogeneous time delays. The novel contribution of this paper lies in the development of new coordinate state transformations, which are used to transform the interconnected subsystems into decoupled subsystems. Most significantly, each decoupled subsystem does not contain any time delay in the state vector. Moreover, each decoupled subsystem is expressed in an observable canonical form, with time delays only appearing in the inputs and outputs of the system. Due to this novel structure, a reducedorder functional observer for each decoupled subsystem can be easily designed to estimate the unmeasurable local state vector. The designed observers for the local subsystems do not need to exchange the state estimates amongst themselves, and therefore, each observer for each local subsystem can be designed independently. Because of the state transformations, the designed observers have a more general structure than any of the existing distributed functional observers available in the literature. Numerical examples are given to illustrate the effectiveness and advantages of our results.


[^19]
# Thuật toán vectơ $\overrightarrow{0}$ giải bài toán quy hoạch tuyến tính dạng chuẩn và ứng dụng 

N. A. Tuấn ${ }^{1}$ và L. Tuấn ${ }^{2}$

Tóm tắt: Như chúng ta đã biết, hầu hết các bài toán quy hoạch tuyến tính trên thực tế đều ở dạng chuẩn tắc, tức là miền ràng buộc của bài toán là hệ bất phương trình tuyến tính. Đồng thời việc biết một điểm chấp nhận được trong miền ràng buộc khi bài toán có số biến lớn là rất khó khăn. Chính vì vậy, trong báo cáo này chúng tôi giới thiệu một số cải tiến theo ý tưởng của phương pháp đơn hình xây dựng một số thuật toán giải các bài toán quy hoạch tuyến tính dạng chuẩn thường gặp với cơ sở xuất phát ban đầu dễ dàng có được bằng nhiều hướng đi đến lời giải là vectơ $\overrightarrow{0}=(0,0, \ldots, 0)$ và trong trường hợp tổng quát là vectơ tựa $\overrightarrow{0}$ (có thể có $n$ vectơ như vậy).

Nhiều bài toán ví dụ bằng số minh hoạ được kiểm nghiệm giải bởi các thuật toán đề nghị này đều được trích lấy từ sách, giáo trình giảng dạy và nhiều tài liệu, công trình trong nước và nước ngoài của các tác giả khác nhau. Kết quả tính toán đi đến lời giải cho thấy hầu hết số bước lặp và số phép tính toán trong mỗi bước lặp đều ít hơn rõ rệt so với việc giải chúng bằng các thuật toán đơn hình hay đơn hình đối ngẫu. Chẳng hạn, bài toán của các tác giả KLEE - MINTY với số chiều là $n$ bất kỳ vẫn cho lời giải sau 1 bước lặp hoặc ngay ở bước chuẩn bị (bước 0 ) của thuật toán.

Thuật toán vectơ $\overrightarrow{0}$ và vectơ tựa $\overrightarrow{0}$ đã được giới thiệu và giảng dạy tại một số lớp đại học (nhất là đối với các trường thuộc khối kinh tế) cho thấy học viên và sinh viên rất thích vì nó không vất vả khi sử dụng làm bài tập như đối với phương pháp đơn hình cổ điển, bởi bảng lặp nón xoay giải khá ngắn gọn và ít bước lặp tính toán hơn trên cùng một bài tập.

Những kết quả này có thể là một gợi ý để chúng ta thêm tin tưởng rằng, trong một tương lai không xa, chúng ta sẽ có thể tìm ra đáp án cho câu hỏi mở của M. J. Todd: "Tồn tại hay không các nguyên tắc quay để cho các biến thể của thuật toán đơn hình có độ phức tạp đa thức?"

[^20]
# Special Palais-Smale's Values and Global Hölderian Error Bound of Polynomial Functions 

H. H. Vui ${ }^{1}$

Abstract: Let $f: \mathbb{R}^{n} \longrightarrow \mathbb{R}$ be a polynomial. For $t \in \mathbb{R}$, put
$[f \leq t]:=\left\{x \in \mathbb{R}^{n}: f(x) \leq t\right\}$
$(f(x)-t)_{+}:=\max \{0, f(x)-t\}$.
We say that $[f \leq t]$ has a global Hölderian error bound (GHEB for short) if there exist $\alpha, \beta, c>o$ such that

$$
(f(x)-t)_{+}^{\alpha}+(f(x)-t)_{+}^{\beta} \geq c \operatorname{dist}(x,[f \leq t])
$$

for all $x \in \mathbb{R}^{n}$.
Let $G(f):=\{t \in \mathbb{R}:[f \leq t]$ has a GHEB $\}$. We show that
i) $G(f)=\mathbb{R}$, if $f$ is a generic polynomial;
ii) In assuming that the set $P S(f)$ of the Palais - Smale values of $f$ is finite, we have

$$
G(f)=[S(f),+\infty] \backslash P S^{1}(f),
$$

where $S(f)$ is the threshold of the global Hölderian error bounds of $f$ and $P S^{1}(f)$ is the set of special Palais-Smale's values of $f$;
iii) If $f$ is a polynomial of 2 variables, then $P S(f)$ is always finite. Hence, the above formula for $G(f)$ holds for any polynomial of 2 variables. Moreover, for this case, $G(f)$ can be computed explicitly and algorithmically.

The arguments of proofs are very elementary, they rely on some basic facts of semi-algebraic sets and functions.

[^21]
# Some Stochastic Optimization Methods for the Posterior Inference Problem in Topic Models 

B. T. T. Xuan ${ }^{1,2}$, T. Q. Khoat $^{1}$, and V. V. Tu ${ }^{1}$

Abstract: In LDA [2], the MAP estimation of topic mixture for a given document $d$ :

$$
\begin{equation*}
\boldsymbol{\theta}^{*}=\arg \max _{\boldsymbol{\theta} \in \bar{\Delta}_{K}} \operatorname{Pr}(\boldsymbol{d}, \boldsymbol{\theta} \mid \boldsymbol{\beta}, \alpha)=\arg \max _{\boldsymbol{\theta} \in \bar{\Delta}_{K}} \operatorname{Pr}(\boldsymbol{d} \mid \boldsymbol{\theta}, \boldsymbol{\beta}) \operatorname{Pr}(\boldsymbol{\theta} \mid \alpha) \tag{1}
\end{equation*}
$$

Under the assumption about the generative process, problem (1) is equivalent to the following: $\boldsymbol{\theta}^{*}=\arg \max _{\boldsymbol{\theta} \in \bar{\Delta}_{K}} \sum_{j} d_{j} \log \sum_{k=1}^{K} \theta_{k} \beta_{k j}+(\alpha-1) \sum_{k=1}^{K} \log \theta_{k}$.

There are approximation methods and sampling methods proposed to solve it. Recently, this estimation problem is considered by many researchers, and many methods such as Variational Bayes (VB) [2], Collapsed Variational Bayes (CVB) [6], CVB0 [1], Collapsed Gibbs Sampling (CGS) [3,4], and Online Maximum a Posteriori Estimation (OPE) [7] have been proposed and applied. However, most of them do not have any clear theoretical guarantee of neither quality nor rate of convergence. The posterior inference problem is in fact nonconvex and is NP-hard [5]. OPE [7] is another approach with concise guarantee on quality and convergence rate, in which we cast the estimation of the posterior distribution into convex/non-convex optimization problem. In this paper, we propose the more general and flexible versions of OPE, so-called G-OPE1, G-OPE2, G-OPE3, which not only enhance the flexibility of OPE in different real-world datasets but also preserve key advantage theoretical characteristics of OPE when comparing to the state-of-the-art methods. We employ G-OPE1, G-OPE2, G-OPE3 as inference method for a topic proportion of a document within large text corpora. The experimental and theoretical results show that our new approaches perform better than OPE and other state-of-the-art methods.

## References

[1] A. Asuncion, M. Welling, P. Smyth, and Y. W. Teh. On smoothing and inference for topic models. In Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, 27-34. AUAI Press, 2009.
[2] D. M. Blei, A. Y. Ng, and M. I. Jordan. Latent dirichlet allocation. Journal of machine Learning research. 3 (2003), 993-1022.
[3] T. L. Griffiths and M. Steyvers. Finding scientific topics. In Proceedings of the National academy of Sciences, 101, 5228-5235, 2004.
[4] M. Hoffman, D. M. Blei, and D. M. Mimno. Sparse stochastic inference for latent dirichlet allocation. In Proceedings of the 29th International Conference on Machine Learning (ICML12), 1599-1606. ACM, 2012.
[5] D. Sontag and D. Roy. Complexity of inference in latent dirichlet allocation. In Neural Information Processing System (NIPS), 2011.
[6] Y. W. Teh, K. Kurihara, and M. Welling. Collapsed variational inference for hdp. In Advances in neural information processing systems, 1481-1488, 2007.
[7] K. Than and T. Doan. Guaranteed inference in topic models. arXiv:1512.03308, 2015.

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