**Some recent progress on selected problems in number theory**

Institute of Mathematics, Hanoi  
23/4/2024 and 25/4/2024

# **GENERAL INFORMATION:**

Time: 23/4/2024 (2pm-4pm) and 25/4/2024 (1pm-3:30pm)  
Location: Room 612 - Building A6, Institute of Mathematics - VAST  
Organizer: Ngô Trung Hiếu (Institute of Mathematics, Hanoi)

# **SCHEDULE OF 23/4/2024:**

14:00-14:30: [Nguyễn Duy Tân](http://sami.hust.edu.vn/giang-vien/?name=tannd) - Hanoi University of Science and Technology  
Some arithmetic properties of Fekete polynomials of principal Dirichlet characters  
  
14:40-15:10: [Đào Phương Bắc](http://mim.hus.vnu.edu.vn/vi/canbo/bacdp) - VNU University of Science in Hanoi  
On the cocharacter closure of rational orbits for algebraic group actions over valued fields  
  
15:20-15:50: [Đào Văn Thịnh](https://www.genealogy.math.ndsu.nodak.edu/id.php?id=251392) - Institute of Mathematics - VAST  
Bhargava-Shankar conjecture over function field

**SCHEDULE OF 25/4/2024:**13:00-13:30: Hà Minh Dũng - Hanoi University of Science and Technology  
Introduction to linear sieves and weighted sieves  
  
13:40-14:10: Hoàng Đức Anh - Institute of Mathematics - VAST  
Richert’s weighted sieve and least almost-primes in arithmetic progressions  
  
14:20-14:50: Hoàng Đức Anh - Institute of Mathematics - VAST  
A weighted sieve of Greaves–Halberstam–Richert and least almost primes in arithmetic progressions

# **ABSTRACTS:**

**Speaker:**[Nguyễn Duy Tân](http://sami.hust.edu.vn/giang-vien/?name=tannd) (Hanoi University of Science and Technology)  
**Title:** Some arithmetic properties of Fekete polynomials of principal Dirichlet characters **Abstract:** In this talk, I will discuss some arithmetic properties of Fekete polynomials associated with principal Dirichlet characters. I will mainly focus on their cyclotomic factors. This is joint work with Shiva Chidambaram, Jan Minac and Tung T. Nguyen.

**Speaker:** [Đào Phương Bắc](http://mim.hus.vnu.edu.vn/vi/canbo/bacdp) (VNU University of Science in Hanoi)  
**Title:** On the cocharacter closure of rational orbits for algebraic group actions over valued fields  
**Abstract:** Let $G$ be a linear algebraic group acting on an affine variety $V$, all are defined over a field $k$. In 2005, G. Roehrle et al have proposed a geometric approach to study completely reducible subgroups due to J.-P. Serre via cocharacter closedness of rational orbits $G(k).v$. Here we say that the rational orbit $G(k).v$ of a rational point $v$ is cocharacter closed if this orbit contains the limit point (if exists) along any cocharacter of $G$. Now assume further that $k$ is a valued field, we may endow $G(k)$ and $V(k)$ with the $v$-adic topology induced from that of the base field $k$. The aim of this talk is to discuss the relationship between the cocharacter closedness and Hausdorff closedness of rational orbits, as well as the cocharacter closure and Hausdorff closure of these ones.

**Speaker:** [Đào Văn Thịnh](https://www.genealogy.math.ndsu.nodak.edu/id.php?id=251392) (Institute of Mathematics - VAST)  
**Title:** Bhargava-Shankar conjecture over function field  
**Abstract:** Over a number field, when all elliptic curves are ordered by height, Manjul Bhargava and Arul Shankar showed that the average size of the n-Selmer group (n<7) is equal to s(n), the sum of the divisors of n. A direct consequence is that there is a density of at least 83.75% elliptic curves having rank 0 or 1, thus, the same amount satisfies the Birch and Swinnerton-Dyer conjecture. Bhargava and Shankar conjectured that the above statement also holds for arbitrary n.

In this talk, I will consider a similar problem but over function fields instead. Precisely, I will confirm the Bhargava-Shankar conjecture over function fields in case n<7. The result will be obtained by a uniform geometric method that could be applied to the general case.

**Speaker:** Hà Minh Dũng(Hanoi University of Science and Technology)  
**Title:** Introduction to linear sieves and weighted sieves  
**Abstract:** This talk is a general introduction to the classical theory of linear sieves and weighted sieves.

**Speaker:** Hoàng Đức Anh (Institute of Mathematics - VAST)  
**Title:** Richert’s weighted sieve and least almost-primes in arithmetic progressions  
**Abstract:** Richert’s weighted sieve is a simple but effective tool to detect almost-primes. In this talk, we discuss a version of Richert’s weighted sieve for almost-primes in arithmetic progressions and several techniques to handle the log-weighted sum in the sieve, including the bilinear form of the error term in the Rosser-Iwaniec linear sieve and a two-dimensional Selberg sieve.

# **Speaker:** Hoàng Đức Anh (Institute of Mathematics - VAST) **Title:** A weighted sieve of Greaves–Halberstam–Richert and least almost primes in arithmetic progressions **Abstract:** The weighted sieve of Greaves–Halberstam–Richert improves upon the traditional weighted sieve of Richert by dropping negative weights and incorporating an error term of bilinear form. In this talk, we discuss how to adapt the Greaves–Halberstam–Richert sieve to arithmetic progressions, including how to handle the bilinear error term. As an application, we prove that in any arithmetic progression a modulo q with gcd(a,q)=1, there is a number asymptotically less than q^{1.8337} that has at most two prime factors, counted with multiplicity. The talks are based on ongoing joint work with Dung Ha and Hieu Ngo.