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IN CLIFFORD ANALYSIS TO A MACH MADE ANALYSIS

TRAN QUYET THANG

Abstract. The main purpose of this paper is to extend some properties of multi-monogenic functions, which are a generalization of monogenic functions in higher dimensions, for a class of functions satisfying Vekua-type generalized Cauchy-Riemann equations in Clifford analysis. It is proved that the Hartogs theorem is valid for these functions.

1. INTRODUCTION

In [1] the theory of functions taking values in a Clifford algebra was studied. It is proved that many important properties of holomorphic functions of one complex variable may be extended to monogenic functions which are solution of the generalized Cauchy-Riemann equations and play an important role in theoretical physics.

Following this way, in [6] Le Hung Son introduced a version of multi-monogenic functions which are a generalization of monogenic functions in higher dimensions and proved some properties of these functions; among them there are the Hartogs extension theorems.

The purpose of this paper is to extend these results for functions satisfying Vekua-type generalized Cauchy-Riemann equations, which are a generalization of multi-monogenic functions for Vekua-type in Clifford analysis. It is proved that the Hartogs extension theorem is valid for these functions. This is a generalization of some results in [4], [5], [6], [7].

2. PRELIMINARIES AND NOTATIONS and and (w.x) a gradient

Let \mathcal{A} be the universal Clifford algebra constructed over a real m-dimensional vector space V with orthonormal basic e_1, e_2, \ldots, e_m (see [1]). A basic for \mathcal{A} is given by

$$\{e_A: A = (h_1, \dots, h_r) \in \mathcal{P}\{1, \dots, m\}, \quad 1 \leq h_1 < \dots < h_r \leq m\},$$

where $e_{\emptyset} = e_0 = 1$ is the identity element.

Multiplication in A is defined by the following rule for the basic elements

und one of ignoral back that
$$e_ie_j + e_je_i = -2\delta_{ij}e_{0}$$
 in $\int_{\mathbb{R}^2} dz = \int_{\mathbb{R}^2} dz = \int_{\mathbb{R}^2}$

For $n \leq m$, \mathbb{R}^{n+1} is naturally imbedded into A. Hence $x = (x_0, x_1, \ldots, x_n) \in \mathbb{R}^{n+1}$ will be identified with

where
$$f_{n+1}(u,x) = (u,x)$$
 and $f_{n+1}(u,x) = (u,x)$ and $f_{n+1}(u,x)$

The involution of x is denoted by

3. If
$$iD_x f(x) = iD_x m$$
 we have finish (left- $\bar{x}_{im} o x = ix$) with respect that (see [1]) a liftor each fixed $y \in G_x f$ is managenic with respect to x and for each fixed $x \in G_x f$

is monogenic with respect to y, then f is defined by leading $D_x f = D_y f_{00} = 0$ (see [6]). The set of multi-monogenic functions on G

is denoted by
$$\mathcal{M}(G)$$
, $\mathcal{M}(G)$, $\mathcal{M}(G$

for the element $a = \sum_{A} a_{A}e_{A}$. Then A turns out to be a Banach algebra of dimension 2^{m} .

In what follows, we shall denote an open subset of the Euclidean space $\mathbb{R}^{n+1} \times \mathbb{R}^{k+1}$ by $G = G_1 \times G_2$ where 1 < n < m, 1 < k < m, n+k < m. We shall consider functions defined on G and taking values in the Clifford algebra A. These functions are given by the following form:

Relation
$$f:G \to A,$$
 for every $k, q \in N$.

$$(\widehat{n}) = L^{2}(\widehat{n}) \times (\widehat{n}) \times (\widehat{n})$$

where $f_A(x,y)$ are functions taking real values.

We introduced the following operators

to replace
$$V$$
, with orthonor and basic e_1, e_2, \dots, e_n (see [1]). A basic for Alisov very $D_x = \sum_{i=0}^n e_i \frac{\partial}{\partial x_i}$, $D_y = \sum_{j=0}^n e_j \frac{\partial}{\partial y_j}$, A basic for Alisov very $D_x = \sum_{i=0}^n e_i \frac{\partial}{\partial x_i}$, $D_y = \sum_{j=0}^n e_j \frac{\partial}{\partial y_j}$, A basic for Alisov very $D_x = \sum_{i=0}^n e_i \frac{\partial}{\partial x_i}$, $D_y = \sum_{j=0}^n e_j \frac{\partial}{\partial y_j}$, $D_y = \sum_{j=0$

$$J_{G_1}f(x,y)=-\int\limits_{G_1}E_{n+1}(u,x)f(u,y)du$$
 for each fixed $y\in G_2$, $J_{G_2}f(x,y)=-\int\limits_{G_2}E_{k+1}(v,y)f(x,v)dv$ for each fixed $x\in G_1$,

where $E_{n+1}(u,x) = \frac{1}{\omega_{n+1}} \frac{\overline{u} - \overline{x}}{|u-x|^{n+1}}$ and ω_{n+1} is the area of the unit sphere in \mathbb{R}^{n+1} . Then the following relation is valid (see [3]):

$$D_x J_{G_1} f(x, y) = f(x, y) \text{ for } f(x, y) \in L^1(G_1).$$
 (1)

For the operators D_y and J_{G_2} the analogous relation also holds.

If $D_x f = 0$, we say f is (left-)monogenic with respect to x (see [1]). If for each fixed $y \in G_2$ f is monogenic with respect to x and for each fixed $x \in G_1$ f is monogenic with respect to y, then f is called multi-monogenic on G (shortly speaking $D_x f = D_y f = 0$) (see [6]). The set of multi-monogenic functions on G is denoted by $\mathcal{M}(G)$.

Following [3], we say that an A-valued function w belongs to $L^{p,s}(R^s)$ if |w| and $|w^{(s)}|$ belong to $L^p(\Delta_s)$ where $w^{(s)}(x) = |x|^{-s}w\left(\frac{1}{x}\right)$ and Δ_S is the unit ball in R^s

The space $L^{p,s}(\mathbf{R}^s)$ is normed by

$$\|A\|_{p,s} = \|A, \Delta_s\|_p + \|A^{(s)}, \Delta_s\|_p.$$

Is what follows, we shall denote an open subset of the Euclidean space

If Ω is a boundary domain of \mathbb{R}^n , we have

$$C^k(\overline{\Omega}) \subset L^q(\overline{\Omega}) \subset L^{p,n}(I\!\!R^n)$$
 for every $k,q \in N$.

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In this paper we shall consider the system

$$\begin{cases} D_x w(x,y) - w(x,y)A(x) &= 0 \text{ for each fixed } y \in G_2, \text{ and equal to the property of the property of$$

$$A \in L^{p,n+1}(\mathbf{R}^{n+1}), \quad B \in L^{q,k+1}(\mathbf{R}^{k+1}), \quad p > n+1, \quad q > k+1.$$
 (3)

theory showed that if the equation (4) (resp. the equation From the Fredholm theory the system (2) relates to the following integral equations

w(c,v) - Truster, v) = oter w

$$w(x,y) - J_{G_1}(w(x,y)A(x)) = 0,$$
 (4)

generalized Cauchy-Riemann equations

The Hence we have the following

Then the function

notition
$$w(x,y) + J_{G_2}(w(x,y)B(y)) = 0$$
 , $x = 1 - (y,x)$. $q_2(5)$

In the next section we need the following known results.

Lemma 1 ([3], Theorem 4.29). Under the assumption (3), the operators $T_1w :=$ $J_{G_1}(wA)$ and $T_2w:=J_{G_2}(wB)$ are compact in the spaces $C(\mathbf{R}^{n+1})$ and $C(\mathbf{R}^{k+1})$ resp., where $C(\mathbf{R}^{n+1})$ denotes the space of bounded continuous functions in \mathbf{R}^{n+1} .

Lemma 2. For a given monogenic function & on G, there exists an unique func-

Furthermore,

$$|T_1 w| \le M|A|_{p,n+1}|w|_{\infty},\tag{6}$$

$$|T_2w| \le N|B|_{q,k+1}|w|_{\infty},\tag{7}$$

where M and N are constants depending on p and q only, and

$$|w|_{\infty} = \sup_{R^{n+1} \times R^{k+1}} |w|.$$

Corollary 1. Assume that

equal to
$$|A| < M^{-1}$$
 and $|B| < N^{-1}$ and $|B| < N^{-1}$

where M and N are the constants in (6) and (7). Then the equations (4) and (5) possess only trival solutions.

In the sequel we only consider solutions of the system (2) in which the given functions A and B satisfy the conditions (3) and (8).

3. GENERALIZED MULTI-MONOGENIC FUNCTIONS

Definition. Under the assumptions (3) and (8), a function $w \in C^1(G)$ is called generalized multi-monogenic if it satisfies the system (2). The system (2) is called generalized Cauchy-Riemann equations.

The set of generalized multi-monogenic functions on G is denoted by $\mathcal{X}(G)$.

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(t)A(v.x)w

The Fredholm theory showed that if the equation (4) (resp. the equation (5)) possesses only the trival solution, the inhomogeneous equation

$$w(x,y)-T_1w(x,y)=\varphi(x,y)$$

(resp. $w(x,y) - T_2w(x,y) = \varphi(x,y)$) possesses an unique solution.

Hence we have the following

Lemma 2. For a given monogenic function φ on G_1 there exists an unique function w which is a C1-solution of the equation

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(44%)) but (44%)) seepen and at the purpose are (8 a), at the first (A w), by the first (A w), by the first (A w) and (9) are the first (
$$w(x)A(x)$$
) = $\varphi(x)$, at (44%)) are the first (9)

where G_1 is a (bounded) simply connected domain with smooth boundary in \mathbb{R}^{n+1}

Conversely, we also have i kiniben 188 Kabebrahi

Lemma 3. Let w be a C^1 -solution on G_1 , continuous on \overline{G}_1 of the equation

$$D_x w(x) - w(x) A(x) = 0.$$

Then the function

$$\varphi(x) = \frac{1}{\omega_{n+1}} \int_{\partial G_1} \frac{\overline{t} - \overline{x}}{|t - x|^{n+1}} d\sigma_t w(t)$$
(10)

is monogenic on G_1 , continuous on \overline{G}_1 , and satisfying the equation (9).

Proof. We observe that the function

$$Tw(x) = -\frac{1}{\omega_{n+1}} \int_{G_1} \frac{\overline{t} - \overline{x}}{|t - x|^{n+1}} w(t) A(t) dt$$

is monogenic in the exterior of \overline{G}_1 and equals to zero at ∞ . The function $E_{n+1}(u,x)$ is also (left- and right-)monogenic with respect to u in the exterior of \overline{G}_1 and equals to zero at ∞ for every $x \in G_1$.

Thus, by Corollary 9.3 in [1], we have

The proof
$$(v,x) = \int_{x} E_{n+1}(u,x) d\sigma_u Tw(u) = 0.$$
 (11)

The proof of the contract that $\partial_{G_1} U(u,x) = 0$ is a second representation of the contract o

We shall above that the lake here

By (1), w(x) - Tw(x) is monogenic on G_1 . Hence we can apply Cauchy's integral formula ([1], Corollary 9.6) to obtain

$$egin{align} w(x)-Tw(x)&=\int\limits_{\partial G_1}E_{n+1}(t,x)d\sigma_tig(w(t)-Tw(t)ig)\ &=\int\limits_{\partial G_1}E_{n+1}(t,x)d\sigma_tw(t)=arphi(x). \end{array}$$

As w is continuous in \overline{G}_1 and Tw is continuous in \mathbb{R}^{n+1} , it follows that φ is continuous in \overline{G}_1 . \square

In view of the preceding results we have

Theorem 1. There is a '1-1' correspondence between the set $M(G) \cap C(\overline{G})$ and the set $M(G) \cap C(\overline{G})$.

Proof. At first, let $\varphi \in \mathcal{M}(G) \cap C(\overline{G})$. For each fixed $y \in G_2$ by Lemma 2 there exists an unique function v which is a C^1 -solution of the equation

$$v(x,y) - J_{G_1}(v(x,y)A(x)) = \varphi(x,y). \tag{12}$$

We observe that v is monogenic with respect to the variable y. Indeed, from (12) it follows that

$$D_y v(x, y) - J_{G_1} D_y (v(x, y) A(x))$$

= $D_y v(x, y) - J_{G_1} (D_y v(x, y)) A(x) = D_y \varphi(x, y) = 0$,

COLLEGE For force but he for an in a state of the force of the force of

and hence, by Corollary 1, $D_y v(x, y) = 0$.

Again using Lemma 2, for each fixed $x \in G_1$ there exists an unique function w stisfying G to rotate add at a page of the page of the stiffying G to rotate add a page of the page of G.

$$w(x,y) - J_{G_2}(w(x,y)B(y)) = v(x,y).$$
 (13)

We shall show that w is generalized multi-monogenic. Indeed, from (13),

$$D_x w(x,y) - J_{G_2}(D_x w(x,y)B(y)) = D_x v(x,y).$$

On the other hand, by (1) and (12),

By (1),
$$w(x) = Tw(x)$$
 demonstance $w(x,y) = v(x,y) + v(x)$ apply Cauchy's integral formula $(x) = [w(x,y) + J_{G_2}(w(x,y)B(y))]$

Thus,

$$\left(D_xw(x,y)-w(x,y)A(x)\right)-J_{G_2}\left(D_xw(x,y)-w(x,y)A(x)\right)B(y)=0$$

and therefore, by Corollary 1, one has

$$D_x w(x,y) - w(x,y)A(x) = 0.$$

Obviously $D_y w(x,y) - w(x,y)B(y) = 0$ (by (13) and (1)) and (1)

Notice that w is clearly continuous in \overline{G} and thereby $w \in \mathcal{X}(G) \cap C(\overline{G})$. Conversely, let $w \in \mathcal{X}(G) \cap C(\overline{G})$. Consider the function

bno
$$(\overline{D}) \cap (\overline{D}) \wedge T_2 w(x,v) = \int_{\mathbb{C}_2} \int_{\mathbb{C}_k+1} (t,v) w(x,t) B(t) dt$$
. From $(\overline{D}) \cap (\overline{D}) \wedge T_2 w(x,v) = \int_{\mathbb{C}_2} \int_{\mathbb{C}_2} E_{k+1}(t,v) w(x,t) B(t) dt$.

By a method analogous to that used to prove Lemma 3 we have

The problem of the moitules
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 and $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are the following problem of $C_{\nu} = C_{\nu}$ and $C_{\nu} = C_{\nu}$ are

Applying Cauchy's integral formula ([1], Corollary 9.6) for the monogenic (with respect to x) function $w(x,y) - T_2w(x,y)$ and using (14) we obtain

$$w(x,y)-T_2w(x,y)=\int_{\mathbb{R}} E_{k+1}(v,y)d\sigma_v\left(w(x,v)\right)_{v,y}d\sigma_v\left(w(x,v)\right)_{v,y}d\sigma_v\left(w(x,v)\right)=0,$$
 $C=(v,x)v_yd=\int_{\mathbb{R}} E_{k+1}(v,v)d\sigma_v\left(w(x,v)\right)=0,$ $C=(v,x)v_yd=0,$ $C=(v,x$

Clearly, q is continuous in \overline{G} and $D_{\nu}g(x,y)=0$. The stands of t In a similar way we also have

$$\int\limits_{\partial G_1} E_{n+1}(u,x) d\sigma_u T_1 w(u,y) = 0 \quad ext{for each } (x,y) \in G$$

"Abortying the Cauchy's integral formula [11], Corollary 9:6) for the menogebia

$$g(x,y) - T_1 g(x,y) = \int\limits_{(u,u)} E_{n+1}(u,x) d\sigma_u \left(g(u,y) - T_1 g(u,y)\right)$$

$$= (u,u) w_u \circ h(x,u) + \delta G_1 \setminus (u,u) \circ h(x$$

Finally, from the identity theorem for the monogenic function (see 11. The It is easy to check that f is multi-monogenic in G and continuous in \overline{G} . This completes the proof of the theorem.

Now we may without restriction assume that Theorem 2. (Uniqueness theorem). If w is generalized multi-monogenic in G and w = 0 in a non-empty open subset $\sigma \subset G_0$ then w = 0 in G.

Proof. We show first that, for each fixed $y \in G_2$, if w is a solution of the equation

For each fixed
$$y \in \sigma_2$$
, $w(\cdot,y)$ is a solution of the equation of the equation $D_x w(x,y) = 0$

send enclosed to you a form for (D, w (x,y)) ? w(x,y) A(x) = 0,8 \cred vood vhe notes and w = 0 in a non-empty open subset $\sigma_1 \subset G_1$, then w = 0 in G_1 . and the equals to zero in $\sigma_1 \times \{y\}$. This implies that w = 0 in gridled, putting in a subject that w = 0 in $\sigma_1 \times \{y\}$.

and the sen arbitrarily in 92. W. F. Win & water and and the series with the series

Next take arbitrary
$$\varphi(x,y)=\int\limits_{-\infty}^{\infty}E_{n+1}(u,x)d\sigma_uw(u,y)$$
 the equation of the author $\varphi(x,y)=\int\limits_{-\infty}^{\infty}E_{n+1}(u,x)d\sigma_uw(u,y)$ by the lines are and Department of the author $\varphi(x,y)=\int\limits_{-\infty}^{\infty}E_{n+1}(u,x)d\sigma_uw(u,y)$

and using Lemma 3 we have

on similar way we also show
$$w=0$$
 in $\{x\}$ in $0=w$ when $x\in \mathbb{R}$ and as $x\in \mathbb{R}$ arbitrarily in G , this yields $w=0$ in G , where G is G in G , this yields $w=0$ in G , the G is G in G . The following theorem is the main results of this paper.

Books Ltd., London, 1982 broken $\partial = w(x,y) - \left(\sum_{n=1}^{\infty} E_{n+1}(t,u)w(t,y)A(t)dt \right)$. Suppose Theorem 2. G2 are domains in Rn+1 and Root foreast, Dat Wiebe an eighbourhood of The second term of this representation is a monogenic function with respect to x in σ_1 . Thus by Cauchy's theorem ([1], Corollary 9.3) we obtain

$$\int_{\partial \sigma_1} E_{n+1}(u,x) d\sigma_u \int_{G_1 \setminus \sigma_1} E_{n+1}(t,u) w(t,y) A(t) dt = 0.$$
 (15)

Applying the Cauchy's integral formula ([1], Corollary 9.6) for the monogenic function φ and using (15) we get

$$\varphi(x,y) = \int_{0}^{\infty} \frac{f(u,y)\partial_{u} d\sigma_{u}(u,y)}{E_{n+1}(u,x)d\sigma_{u}\varphi(u,y)} = \int_{0}^{\infty} \frac{f(u,x)\partial_{u} f(u,y)}{E_{n+1}(u,x)d\sigma_{u}w(u,y)} = 0$$

$$(u,x)^{2} = (u,u)w_{u}ob(u,u) + 3 \frac{\partial \sigma_{u}}{\partial u}(u,u) + 3 \frac{\partial \sigma_{u}}{\partial u}(u,u) = 0$$

for each $x \in \sigma_1$.

Finally, from the identity theorem for the monogenic function (see [1], Theorem 11.3.9) we conclude that $\varphi(\cdot,y)=0$ in G_1 and, therefore, by Corollary 1, $w(\cdot,y)=0$ in G_1 .

Now we may without restriction assume that

Theorem 2. (Uniqueness theorem). If w is generalized multi-monogenic in
$$G$$
 and $w=0$ in a non-empty open $u, k_0 \times \sigma_1 G = \sigma$ then $w=0$ in G .

where σ_1 and σ_2 are polydisks in R^{n+1} and R^{k+1} resp. Land that work and look

For each fixed $y \in \sigma_2$, w(.,y) is a solution of the equation $0 = (x) \wedge (y, z) = (y, z) \otimes (x) \otimes (y, z) = 0$

$$D_x w(x,y) - w(x,y)A(x) = 0,$$

and equals to zero in $\sigma_1 \times \{y\}$. This implies that w = 0 in $G_1 \times \{y\}$ and as y is chosen arbitrarily in σ_2 , w = 0 in $G_1 \times \sigma_2$.

Next take arbitrary $x \in G_1$, then w(x, .) is a solution of the equation

$$D_y w(x,y) - w(x,y)B(y) = 0.$$

In a similar way we also show w=0 in $\{x\}\times G_2$ and as x is also chosen arbitrarily in G_1 , this yields w=0 in $G_1\times G_2$. \square

The following theorem is the main results of this paper.

Theorem 3. (Hartogs extension theorem). Let $G = G_1 \times G_2$ where G_1 and G_2 are domains in \mathbb{R}^{n+1} and \mathbb{R}^{k+1} resp.. Let Σ be an open neighbourhood of

dG. Then for every generalized multi-monogenic function win Σ, there exists an unique generalized multi-monogenic function W in $G \cup \Sigma$ such that W = w in Σ . Le Hung Son, Extension problem for functions with values in a Chifford algebra, Arch. Math. 55

Proof. Without loss of generality we may assume that the given generalized multimonogenic function w is continuous in $\overline{\Sigma}$. Then from Theorem 1 there exists an unique function φ multi-monogenic in Σ and continuous in $\overline{\Sigma}$.

Of STOI to image 19 produce brother a cratsmare discussional acceptant and By [6, Theorem 4.1], there exists an unique multi-monogenic function Φ in $G \cup \Sigma$ such that $\Phi = \varphi$ in Σ for Ω where Ω is Ω is Ω in Ω in Ω in Ω is Ω in Ω

Using Theorem 1 it follows that there exists an unique generalized multimonogenic function W satisfying the system Department of Mathematics Revised October 15, 1995

$$\begin{cases} v(x,y) - J_{G_1}(v(x,y)A(x)) &= \varphi(x,y) & \text{for each fixed } y \in G_2, \text{ in } G_$$

where v is defined as in Theorem 1.

Author Departement of Mathematics Both W and w satisfy the system (16) in Σ . From Corollary 1 and the Fredholm theory it follows W=w in Σ .

Finally, by Theorem 2, W is the unique extension, and the proof is complete.

Remark. We observe that the condition (8) may be replaced by a larger one. In fact, it suffices to require that the functions A and B are given such that the integral equations In order joy say the firme needed for review create sheath line

$$v(x,y)-J_{G_1}ig(v(x,y)A(x)ig)=0 \quad ext{for each fixed } y\in G_2, \ w(x,y)-J_{G_2}ig(w(x,y)B(y)ig)=0 \quad ext{for each fixed } x\in G_1,$$

possess only trival solutions. Then the above results remain valid.

Aknowledgements. The author would like to thank Dr. Le Hung Son and Dr. Dang Khai for their advice and encouragements.

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Using Theorem 1 it follows that there exists an unique generalized multi-

Department of Mathematics metrys and an Received September 24, 1992 Hanoi Polytechnic University Revised October 15, 1993

Dai Co Viet Street does not (y,x)y = -(x,y)(x)A(y,x)y, of -(y,x)y $(W(x,y) - J_{G_x}(W(x,y)B(y)) = v(x,y)$ for each fixed man (W(x,y) + W(x,y)) = v(x,y)and

where v is defined as in Theorem I. Departement of Mathematics and Vinh Pedagogical University (11) mester and visites whom we do to Fredholm theory it follows W = w in E. Vinh. Vietnam

Finally, by Theorem 2, W is the unique extension, and the proof is com-Dete D

Remark. We observe that the condition (8) may be replaced by a larger one. In fact, it suffices to require that the functions A and B are given such that the integral equations and super out to not more and type to the forthcoming issues

send enclose $v(x,y) = J_{\infty}(v(x,y))A(x)) = 0$ (for each fixed $v \in G_2$ and the notice and then fill $(x_1y)_{n,0}$ lead $(x_2y)_{n,0}$ by for each fixed $x \in G$ by reach it

possess only trival solutions. Then the above results remain valid its itidae resouls Next take arbitrary reach, then wir. I are solver of the equation

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