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ON BROWN – McCOY Γ–RINGS

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Abstract. *Let M be a Γ–ring in the sense of Nobusawa. The ring $M_2 = \begin{pmatrix} R & \Gamma \\ M & L \end{pmatrix}$ was defined by Kyuno. In this paper, Brown – McCoy Γ–rings are defined to be those Γ–rings in which the prime radical equals the Brown – McCoy radical in all homomorphic images. The aim of the paper is to study the properties of Brown – McCoy Γ–rings. The relationships between Brown – McCoy properties of Γ–ring M and the corresponding properties of $\Gamma_{n,m}$–ring $M_{m,n}$, the right operator ring R of Γ–ring M, M–ring Γ and the ring M_2 are established.*

1. INTRODUCTION

Throughout this paper, "Γ–ring" means a Γ–ring in the sense of Barnes, "$I \trianglelefteq M$" will denote "I is an ideal of M". For the notions of operator rings, matrix Γ–rings, weak Γ_N–ring, Γ_N–ring etc. we refer to [1], [3], [5] and [6].

The class of Jacobson rings, which consists of all rings in which the prime radical coincides with the Jacobson radical in all homomorphic images, is an important class in ring theory. For several years, Jacobson rings and their generalizations are extensively studied (cf. [11], [12] and [13] etc.). In [13] some of the Jacobson properties have been established for Γ–rings. However, another possible generalization of the Jacobson property is to consider the class of all rings in which, for every homomorphic image, the prime radical is coincident with the Brown – McCoy radical (see [12]). We shall call this the class of Brown – McCoy rings. In this paper, we use the Brown – McCoy radical of Γ–rings in the sense of Booth (see [4]) to study the Brown – McCoy properties of Γ–rings. When (M, Γ) is a Γ_N–ring there are five related rings: the Γ–ring M, the right operator ring R of Γ–ring M, the matrix $\Gamma_{n,m}$–ring $M_{m,n}$, the M–ring Γ and the ring M_2. In this paper the relationships between the Brown – McCoy properties of these rings are established.

A Γ–ring M is said to have right unity (resp. strong right unity) if there exists an element $\sum_{i=1}^{n} [\delta_i, a_i] \in R$ (resp. $[\delta, a] \in R$) such that

$\sum_{i=1}^{n} x\delta_i a_i = x$ (resp. $x\delta a = x$) for all $x \in M$. It is easily verified that in this case, $\sum_{i=1}^{n}[\delta_i, a_i]$ (resp. $[\delta, \alpha]$) is the unity of the ring R. Left unity is similarly defined. An ideal I of M is called modular (resp. right modular, right strongly modular) if the fact Γ–ring M/I has right and left unities (resp. a right unity, a right strong unity). Left modular ideals are similarly defined. The Γ–ring M is called simple if $M\Gamma M \neq 0$ and M has no ideals other than 0 and M.

Let (M, Γ) be a weak Γ_N–ring and $P \subseteq R$, $Q \subseteq L$ and $\Phi \subseteq \Gamma$, then we define

$$P^* = \{x \in M : [\beta, x] \in P \text{ for all } \beta \in \Gamma\},$$
$$Q^+ = \{x \in M : [x, \mu] \in Q \text{ for all } \mu \in \Gamma\},$$
$$A^{*\prime} = \{r \in R : Mr \subseteq A\}, \quad A^{+\prime} = \{y \in L : yM \subseteq A\},$$
$$\Gamma(A) = \{\mu \in \Gamma : M\mu M \subseteq A\} \text{ and } M(\Phi) = \{x \in M : \Gamma x\Gamma \subseteq \Phi\}.$$

For further details of Γ–rings and their operator rings, we refer to [1] and [6].

2. BROWN – McCOY Γ–RINGS AND MATRIX $\Gamma_{n,m}$–RINGS

Radical classes of Γ–rings, special radical and hereditary classes are defined exactly as for rings. See, for example, [3] or [10].

For an arbitrary Γ–ring M, Booth [2] defined the Brown – McCoy radical $B(-)$ to be the upper radical defined by the class of simple Γ–rings having strong left unities. It is shown that the Brown – McCoy radical is a special radical and a generalization of the Brown – McCoy radical of ring. Throughout this paper $B(-)$ and $P(-)$ will denote the Brown – McCoy and prime radicals, respectively.

Definition 2.1. Γ–ring M is called a Brown – McCoy Γ–ring, if the prime radical equals the Brown – McCoy radical in all homomorphic images of M.

From the definition, it is easy to prove the following:

Proposition 2.2. *If M is a Brown – McCoy Γ–ring and $I \trianglelefteq M$, then M/I is also Brown – McCoy.*

Proposition 2.3. *M is a Brown-McCoy Γ-ring if and only if every prime ideal of M is an intersection of maximal left strongly modular ideals of M.*

Corollary 2.4. *Γ-ring M is Brown-McCoy if and only if every prime homomorphic image of M is B-semisimple.*

If we denote by Θ the class of all Brown-McCoy Γ-rings, then, from Definition 2.1, we have

$$\Theta = \{M : B(M/I) = P(M/I) \text{ for every ideal } I \text{ of } M\}.$$

Analogously to [9, Theorem 26] we have that Θ is a radical class. If we denote by λ the radical associated with the Brown-McCoy Γ-ring class Θ, then Θ is the largest radical class such that $\lambda(M) \cap B(M) \subseteq P(M)$ for every Γ-ring M.

Proposition 2.5. *λ is hereditary, i. e. ideals of Brown-McCoy Γ-rings are Brown-McCoy.*

Proof. Let the Γ-ring M be Brown-McCoy and let I be an ideal of M. If I is not Brown-McCoy, then there is a prime ideal K of I such that I/K has a nonzero Brown-McCoy radical, say A/K. Now K will be an ideal of M, since K is a prime ideal of I, so consider the Γ-ring M/K. In this quotient, the prime radical coincides with the Brown-McCoy radical, which contains A/K. Thus A/K is a lower strong nil radical Γ-ring and so contains a nonzero strong nilpotent ideal. This contradicts the primeness of K in I. Therefore I is a Brown-McCoy Γ-ring and the proposition is proved.

Corollary 2.6. *Let M be a Γ-ring and I an ideal of M, then $\lambda(I) = I \cap \lambda(M)$.*

Proposition 2.7. *M is a simple Γ-ring with a strong left unites, then, if $m \leq n$, $M_{m,n}$ is a simple $\Gamma_{n,m}$-ring with a strong left unity.*

Proof. Suppose that M is simple Γ-ring with a strong left unity. Then, by [5, Lemma 3], it is easy to prove that $M_{m,n}$ is a simple $\Gamma_{n,m}$-ring. Let $a \in M$ and $\beta \in \Gamma$ be such that $a\beta x = x$ for all $x \in M$. Then, if $A \in M_{m,n}$, $(a\delta_{i,j}E_{i,j})(\beta\delta_{k,l}E_{k,l})A = A$, where $\delta_{i,j} = 1$, if $i = j$, otherwise 0. If follows that $M_{m,n}$ is a simple $\Gamma_{n,m}$-ring with a strong left unity.

Theorem 2.8. *If M is a Γ-ring and $m \leq n$, then $B(M_{m,n}) \leq (B(M))_{m,n}$.*

Proof. By Proposition 2.7, [6, Lemma 3] and [4, Theorem 4.2], it is easily verified that $B(M_{m,n}) \leq (B(M)_{m,n})$.

Remark 2.9. The Theorem 2.8 does not hold if $m > n$. For example, let F be a field , $M = F$ and $\Gamma = F$. It is easily shown that $B(M) = 0$ and $B(M_{3,2}) = M_{3,2}$. Even if $m \leq n$ it does not always hold that $B(M_{m,n}) = (B(M)_{m,n})$. For example, see [2], p.75.

Corollary 2.10. *Let R be a ring. Then for any $m \leq n$, $B(R_{m,n}) \leq (B(R))_{m,n}$ where $B(R)$ is the Brown-McCoy radical of ring R.*

Corollary 2.11. *Let R be a ring. Then for any n, $B(R_n) = (B(R))_n$, where $B(R)$ is the Brown-McCoy radical of ring R.*

The proof of the above corollary is dependent on the fact that the Brown-McCoy radical of the ring R is equal the Brown-McCoy radical of the Γ-ring with $\Gamma = R$.

Theorem 2.12. *If Γ-ring M is Brown-McCoy and $m \leq n$, then the matrix $\Gamma_{n,m}$-ring $M_{m,n}$ is Brown-McCoy.*

Proof. Suppose that M is a Brown-McCoy Γ-ring. By [6], Theorem 2, any prime ideal of $M_{m,n}$ is of the form $P_{m,n}$, where P is a prime ideal of the Γ-ring M. Since the Γ-ring M is Brown-McCoy and so $P = \cap P_\alpha$, where P_α $(\alpha \in A)$ are maximal left strongly modular ideals of M. Now $P_{m,n} = (\cap P_\alpha)_{m,n} = \cap (P_\alpha)_{m,n}$, and $M_{m,n}/(P_\alpha)_{m,n} \cong (M/P_\alpha)_{m,n}$. By Proposition 2.7, $(P_\alpha)_{m,n}$ is a maximal left strongly modular ideal of $M_{m,n}$. Thus $M_{m,n}$ is a Brown-McCoy $\Gamma_{n,m}$-ring.

Corollary 2.13. *A ring R is a Brown-McCoy ring if and only if, for any n, the R_n-ring R_n is Brown-McCoy, i.e. the matrix ring $M_n(R)$ is a Brown-McCoy ring.*

Corollary 2.14. $\lambda(M_{m,n}) \geq (\lambda(M))_{m,n}$ *for any Γ-ring M and $m \leq n$.*

Proof. This follows immediately from Theorem 2.12 and the corresponding result in [9, Lemma 8].

3. BROWN-McCOY Γ-RINGS AND THE OPERATOR RINGS

In this section, the relationships between Brown-McCoy properties of Γ-ring M and its left operator rings are established. Analogous results for the right operator rings can be proved similarly.

Lemma 3.1. *If A is an ideal of the Γ-ring M, L and $[M/A,\Gamma]$ are the left operator rings of Γ-ring M and Γ-ring M/A, respectively, then we have $[M/A,\Gamma] \cong L/A^{+'}$ under the mapping*

$$\sum_i [x_i + A, \gamma_i] \to \sum_i [x_i, \gamma_i] + A^{+'}.$$

Lemma 3.2. *If M is Γ-ring with left operator ring L, and M has a left unity, P is an ideal of L, $[M/P^+,\Gamma]$ is the left operator ring of Γ-ring M/P^+, then we have $[M/P^+,\Gamma] \cong L/P$ under the mapping*

$$\sum_i [x_i + P^+, \gamma_i] \to \sum_i [x_i, \gamma_i] + P.$$

Lemma 3.3. *If Γ-ring M has a left unity or Γ-ring M is prime, then M is simple if and only if the left operator ring L of M is a simple ring.*

The proof of Lemma 3.1, 3.2, and 3.3 may easily be verified by direct computation.

Lemma 3.4. *Let M be a Γ-ring with left operator rings L.*

(1) *If P is a maximal modular ideal of L, then P^+ is a maximal left modular ideal of M and $(P^+)^{+'} = P$.*

(2) *If Q is a maximal left strongly modular ideal of M, then $Q^{+'}$ is a maximal modular ideal of L and $(Q^{+'})^+ = Q$.*

Proof. (1) Suppose P is a maximal modular ideal of L. Then there exists $e \in L$ such that for all $\alpha \in \Gamma$ and $x \in M$, $e[x,\alpha] - [x,\alpha] \in P$, i.e. $[ex - x, \alpha] \in P$, that is, $ex - x \in P^+$. Hence, by Lemma 3.2 and 3.3, P^+ is a maximal left modular ideal of M. By [6], Theorem 1, and the fact that maximal (left) modular ideals are prime ideals, it is clear that $(P^+)^{+'} = P$.

(2) By Lemma 3.1. and 3.3, the proof is clearly.

Theorem 3.5. *Let M be a Γ-ring and L be the left operator ring of M. Then,*

(1) *If M is a Brown–McCoy Γ-ring then L is a Brown–McCoy ring; and*

(2) *If M has a strong left unity and L is a Brown–McCoy ring, then M is a Brown–McCoy Γ-ring.*

Proof. (1) Suppose that M is a Brown–McCoy Γ-ring. For every prime ideal Q of L, by [8], Theorem 1, there is a prime ideal P of M such that $Q = P^+$. Since M is a Brown–McCoy Γ-ring, there exist maximal left strongly modular ideals P_α of M $(\alpha \in \Lambda)$ such that $P = \cap\{P_\alpha : \alpha \in \Lambda\}$. But

$$\begin{aligned} P^{+'} &= \{x \in R : Mx \subseteq P\} \\ &= \cap\{x \in R : Mx \subseteq P_\alpha,\ \alpha \in \Lambda\} = \cap\{P_\alpha^{+'} : \alpha \in \Lambda\}. \end{aligned}$$

By Lemma 3.4, $P_\alpha^{+'}$ $(\alpha \in \Lambda)$ are maximal modular ideals of L. Therefore, L is a Brown–McCoy ring.

(2) Suppose now that L is a Brown–McCoy ring, and let P be a prime ideal of Γ–ring M, then, by [6], Theorem 1, there is a prime ideal A of L such that $P = A^+ = \{x \in M : [\Gamma, x] \subseteq A\}$. Since L is a Brown–McCoy ring, then $A = \cap\{A_\alpha : \alpha \in \Lambda\}$, where A_α $(\alpha \in \Lambda)$ are the maximal modular ideals of L. But

$$\begin{aligned} P = A^+ &= \{x \in M : [\Gamma, x] \subseteq A\} \\ &= \cap\{x \in M : [\Gamma, x] \subseteq A_\alpha,\ \alpha \in \Lambda\} = \cap\{A_\alpha^+ : \alpha \in \Lambda\}. \end{aligned}$$

By Lemma 3.4 and M has a strong left unity and [6], Theorem 1, A_α^+ $(\alpha \in \Lambda)$ are maximal left strongly modular ideals of M. Hence, M is a Brown–McCoy Γ–ring. This completes the proof.

In the following, let β be the radical class of Brown–McCoy rings.

Theorem 3.6. *Let M be a Γ–ring with left strong unity and L be the left operator ring of M. Then, we have*

(1) $\beta(L) \subseteq (\lambda(M))^{+'}$; and

(2) $\lambda(M) \subseteq (\beta(L))^+$.

Proof. (1) Since $[M/\lambda(M), \Gamma] \cong L/(\lambda(M))^{+'}$ by Lemma 3.1 and $M/\lambda(M)$ is λ–semisimple. Hence by Theorem 3.5, $L/(\lambda(M))^{+'}$ is β–semisimple. It follows that $\beta(L) \subseteq (\lambda(M))^{+'}$.

(2) Since, by Lemma 3.2, $[M/(\beta(L))^+, \Gamma] \cong L/\beta(L)$ and $L/\beta(L)$ is β–semisimple. Then, by Theorem 3.5, it follows that $\lambda(M) \subseteq (\beta(L))^+$.

Corollary 3.7. *Let M be a Γ–ring with left strong and right unities, L be the left operator ring of M. Then, we have $\beta(L) = (\lambda(M))^{+'}$.*

Proof. Since Γ–ring M has right and left unities, it is easy to prove that $[\lambda(M), \Gamma] = (\lambda(M))^{+'}$ and $(\beta(L))^+ = \beta(L)M$. Thus, we have

$$(\lambda(M))^{+'} \subseteq ((\beta(L))^+)^{+'} = (\beta(L)M)^{+'} = [M\beta(L), \Gamma] \subseteq \beta(L).$$

By Theorem 3.6, we have $\beta(L) = (\lambda(M))^{+'}$.

The following examples, given to me by Professor G. L. Booth, show that the above results are false if Γ-ring M does not has strong left unity.

Examples 3.8. Let $m > n$, and let $M = \Re_{m,n}$, $\Gamma = \Re_{n,m}$, where $\Re$ is the real field. Then M is a simple Γ-ring which has no strong left unity, but it does have both left and right unities. It follows that $P(M) = 0$, $B(M) = M$, whence $\lambda(M) = 0$. The left operator ring L of M is isomorphic to $\Re_{m,m}$ and so $\beta(L) = L$. Thus $(\lambda(M))^{+'} = 0 \neq \beta(L)$.

Example 3.9. Let U and V be respectively finite dimensional and countably infinite dimensional vector spaces over the real field $\Re$. Let $M = L(U,V)$ (the set of all linear mappings from U to V, mappings acting on the left), $\Gamma = L(V,U)$. Then M is a Γ-ring with the operations of pointwise addition and composition of functions. The right operator ring R is isomorphic to $L(U,U)$. Moreover, M is simple, and has a right, but not a left unity. Hence, $P(M) = 0$, $B(M) = M$, whence $\lambda(M) = 0$. Thus $(\lambda(M))^{*'} = 0^{*'} = 0$. But $\beta(L) \neq 0$.

4. BROWN-McCOY PROPERTY OF M-RING Γ AND THE RING M_2

In this section, let (M,Γ) be Γ_N-ring, and let R and L denote the right and left operator rings of Γ-ring M, respectively.

The set $M_2 = \begin{pmatrix} R & \Gamma \\ M & L \end{pmatrix}$ is a ring with respect to the obvious operations of matrix multiplication and addition. For details, see [1,8]. Moreover, if $I \trianglelefteq M$, then it is easily verified that

$$I_2 = \begin{pmatrix} I^{*'} & \Gamma(I) \\ I & I^{+'} \end{pmatrix} \trianglelefteq M_2 .$$

Theorem 4.1. *Suppose (M,Γ) is a weak Γ_N-ring and Γ-ring M has a strong left unity. Then the Γ-ring M is Brown-McCoy if and only if the M-ring Γ is Brown-McCoy.*

Proof. Suppose that Γ-ring M is Brown-McCoy. For every prime ideal Φ of M-ring Γ, by [1], Theorem 3.3, there is a prime ideal P of Γ-ring M such that $\Phi = \Gamma(P)$. Hence, there exist maximal left strongly modular ideals P_α of M $(\alpha \in \Lambda)$ such that $P = \cap\{P_\alpha : \alpha \in \Lambda\}$. But

$$\Gamma(P) = \{\gamma \in \Gamma : M\gamma M \subseteq P\}$$
$$= \cap\{\gamma \in \Gamma : M\gamma M \subseteq P_\alpha,\ \alpha \in \Lambda\} = \cap\{\Gamma(P_\alpha) : \alpha \in \Lambda\}.$$

By [1], Theorem 5.1, $\Gamma(P_\alpha)$ $(\alpha \in \Lambda)$ are maximal right strongly modular ideals of Γ. Thus, M-ring Γ is Brown - McCoy. The proof of the converse is similar. This completes the proof.

Proposition 4.2. *Let (M,Γ) be a weak Γ_N-ring. Then $\lambda(M) \subseteq M(\lambda(\Gamma))$ and $\lambda(\Gamma) \subseteq \Gamma(\lambda(M))$, where $\lambda(M)$ and $\lambda(\Gamma)$ denote, respectively, the λ-radical of Γ-ring M and M-ring Γ.*

By Theorem 4.1, and since the fact $(M/\lambda(M), \Gamma/\Gamma(\lambda(M)))$ is weak $(\Gamma/\Gamma(\lambda(M)))_N$-ring, it is clear that $\lambda(\Gamma) \subseteq \Gamma(\lambda(M))$. Similarly, it can be shown that $\mu(M) \subseteq M(\mu(\Gamma))$.

Theorem 4.3. *Let (M,Γ) be a Γ_N-ring and Γ-ring M has a strong left unity. Then the ring $M_2 = \begin{pmatrix} R & \Gamma \\ M & L \end{pmatrix}$ is a Brown - McCoy ring if and only if Γ-ring M is Brown - McCoy.*

Proof. Suppose that M_2 is a Brown - McCoy ring and I is a prime ideal. Hence, by [1], Theorem 3.6, $I_2 = \begin{pmatrix} I^{*\prime} & \Gamma(I) \\ I & I^{+\prime} \end{pmatrix}$ is a prime ideal of M_2. Thus, $I_2 = \cap P_{\alpha 2}$, where $P_{\alpha 2}$ $(\alpha \in \Lambda)$ are maximal modular ideals of M_2. By [1], Theorem 5.3, $P_{\alpha 2} = \begin{pmatrix} P_\alpha^{*\prime} & \Gamma(P_\alpha) \\ P_\alpha & P_\alpha^{+\prime} \end{pmatrix}$, where P_α is a maximal modular ideal of Γ-ring M. From this, we have that $I = \cap P_\alpha$, by the condition, every ideal of M is left strongly modular. Hence M is a Brown - McCoy Γ-ring.

The proof of the converse is similar, and will be omitted.

Theorem 4.4. *Let (M,Γ) be a Γ_N-ring and the Γ-ring M has a strong left unity. Then $\beta(M_2) \subseteq \begin{pmatrix} (\lambda(M))^{*\prime} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+\prime} \end{pmatrix}$*

Proof. Since

$$M_2 / \begin{pmatrix} (\lambda(M))^{*\prime} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+\prime} \end{pmatrix} \cong \begin{pmatrix} R/(\lambda(M))^{*\prime} & \Gamma/\Gamma(\lambda(M)) \\ M/\lambda(M) & L/(\lambda(M))^{+\prime} \end{pmatrix},$$

by Theorem 4.3 and the facts that

$$[\Gamma/\Gamma(\lambda(M)), M/\lambda(M)] \cong R/(\lambda(M))^{*\prime}$$

and $$[M/\lambda(M), \Gamma/\Gamma(\lambda(M))] \cong L/(\lambda(M))^{+\prime},$$

it follows that $\beta(M_2) \subseteq \begin{pmatrix} (\lambda(M))^{*'} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+'} \end{pmatrix}$.

Theorem 4.5. *Let (M,Γ) be a Γ_N-ring and Γ-ring M have right and strong left unities. Then we have*

$$\beta(M_2) = \begin{pmatrix} (\lambda(M))^{*'} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+'} \end{pmatrix}$$

Proof. Since Γ-ring M has right and strong left unities, we can prove that for any $A \triangleleft M$, $A^{*'} = [\Gamma, A]$, $A^{+'} = [A, \Gamma]$ and $\Gamma(A) = \Gamma A \Gamma$ and thus

$$\begin{pmatrix} (\lambda(M))^{*'} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+'} \end{pmatrix} = \begin{pmatrix} [\Gamma, \lambda(M)] & \Gamma\lambda(M)\Gamma \\ \lambda(M) & [\lambda(M), \Gamma] \end{pmatrix}.$$

By [8], Lemma 4.1, we have $\beta(M_2) = \begin{pmatrix} [\Gamma, A] & \Gamma A \Gamma \\ A & [A, \Gamma] \end{pmatrix}$, where $A \trianglelefteq M$. On the other hand, by the fact that

$$M_2/\beta(M_2) = M_2/\begin{pmatrix} [\Gamma, A] & \Gamma A \Gamma \\ A & [A, \Gamma] \end{pmatrix} \cong \begin{pmatrix} R/A^{*'} & \Gamma/\Gamma(A) \\ M/A & L/A^{+'} \end{pmatrix}$$

and since $M_2/\beta(M_2)$ is β-semisimple, from Theorem 4.3, we get that M/A is λ-semisimple and thus $\lambda(M) \subseteq A$. Hence, we have

$$\begin{pmatrix} (\lambda(M))^{*'} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+'} \end{pmatrix} \subseteq \begin{pmatrix} [\Gamma, A] & \Gamma A \Gamma \\ A & [A, \Gamma] \end{pmatrix} = \beta(M_2).$$

Thus, we have $\beta(M_2) = \begin{pmatrix} (\lambda(M))^{*'} & \Gamma(\lambda(M)) \\ \lambda(M) & (\lambda(M))^{+'} \end{pmatrix}$. The proof is completes.

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