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Finite Element Methods for the Equations of Waves in Fluid-Saturated Porous Media

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Abstract. In this paper, finite element methods for the problems of wave propagation in a fluid-saturated porous medium are discussed. The medium is composed of a porous elastic solid (soil, rock, etc.) saturated by a compressible viscous fluid (oil, water, etc.), and the fluid may flow relatively to the solid. Biot's lowfrequency dynamic equations are chosen to describe the problems mentioned above, with stress-given boundary conditions, ABCs (Absorbing Boundary Conditions) on artificial boundaries and conditions on interfaces between the fluid-saturated porous medium and elastic solids.

In the paper, a new kind of discrete ABCs is presented, and a discrete-time Galerkin method are utilized for obtaining approximate solutions. The numerical results show that they both are effective. Two dilatational waves (fast wave P1 and slow wave P2) and one rotational wave (S wave) are clearly visible in the figures of computational results, which coincide with theoretical analysis very well.

1. Introduction

In this paper, the problems of wave propagation in a fluid-saturated porous medium are discussed. The medium is composed of a porous elastic solid (soil, rock, etc.) saturated by a compressible viscous fluid (oil, water, etc.), and the fluid may flow relatively to the solid. This medium model can be used for the seismic wave propagation in reservoir rocks of oil, dams and their bases and so on. A theory of propagation of stress waves in such media has been established by Biot [1, 2], in which the microcosmic motion of the fluid in the pores is not studied, but only the macroscopic motion of the solid-fluid aggregate is considered. It is an amendment of elastic dynamic theory for complex media including fluid.

In fact, in the paper a more general case is considered, i.e., the medium is a composite system consisting of an elastic solid and a fluid-saturated porous medium. An elastic solid with an imbedded fluid-saturated porous medium is an example of such case, which is often met in practice.

The emphasis of the paper is laid on the problems of infinite media. For such problems, it is necessary to introduce artificial boundaries to limit areas of computation and then some ABCs (Absorbing Boundary Conditions) have to be imposed on these boundaries. Because the wave equations of fluid-saturated porous media are complicated, their effective ABCs have not been seen yet. In this paper, a kind of discrete ABCs developed by the author and her cooperator is presented, which can be applied to both elastic wave and fluid-saturated porous medium wave equations. For these two equations, the formulas of the ABCs are the same, but only values of a parameter in them are different, so that it is convenient to be used for the problems discussed in this paper.

The organization of the paper is as follows. In Sec. 2 the mathematical model is described. Sec. 3 concerns the ABCs. In Sec. 4 the weak form of the problem is derived and a discrete-time Galerkin method is defined. In Sec. 5 the numerical results are exhibited, which illustrated the effectiveness of the ABCs and the numerical method suggested in this paper.

For the simplicity of writing, only two dimensional case is discussed.

2. Mathematical Model

Consider a bounded domain $\Omega \subset R^2$ with a piecewise smooth boundary $\partial\Omega$. Let $\Omega = U_1 \cup U_2$ and $U_1 \cap U_2 = \emptyset$, Ω_1 and Ω_2 be the fluid-saturated porous medium and the elastic solid parts, respectively. Suppose that $\partial\Omega_1 = \Gamma_{1s} \cup \Gamma_{1a} \cup \Gamma_{12}$ and $\partial\Omega_2 = \Gamma_{2s} \cup \Gamma_{2a} \cup \Gamma_{1s}$ and Γ_{2s} are stress-given boundaries, Γ_{1a} and Γ_{2a} are artificial boundaries with ABCs, and Γ_{12} is an interface between the two media.

When $\Gamma_{1s} = \Gamma_{1a} = \emptyset$, it is the case that a fluid-saturated porous medium is imbedded in an elastic solid.

Let $u(x,y,t) = (u_x(x,y,t), u_y(x,y,t))^T$ be the displacement vector of the solid in both Ω_1 and Ω_2, and $U(x,y,t) = (U_x(x,y,t), U_y(x,y,t))^T$ be the displacement vector of the fluid in pores in Ω_1. Replacing U, consider the displacement vector w of the fluid relative to the solid, which is defined as

$$w = \beta(U - u),$$

where β is the porosity. Take $W = (u_x, u_y, w_x, w_y)^T$ as the displacement vector of the solid-fluid aggregate.

From the dynamic equations of the fluid-saturated porous media and the Darcy's law satisfied by flow through porous media, the equations in Ω_1 are

$$B_1^T\Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) D_1 B_1 \Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) W = Q_1 \frac{\partial^2 W}{\partial t^2} + Q_2 \frac{\partial W}{\partial t} + F_1, \quad (x,y,t) \in \Omega_1 \times (0,T). \tag{2.1}$$

Here B_1, D_1, Q_1 and Q_2 are the following matrices

$$B_1\Big(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\Big)=\begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} \end{pmatrix},$$

$$D_1=\begin{pmatrix} \lambda_c+2\mu_b & \lambda_c & 0 & -\alpha M \\ \lambda_c & \lambda_c+2\mu_b & 0 & -\alpha M \\ 0 & 0 & \mu_b & 0 \\ -\alpha M & -\alpha M & 0 & M \end{pmatrix},$$

$$Q_1=\begin{pmatrix} \rho & 0 & \rho_f & 0 \\ 0 & \rho & 0 & \rho_f \\ \rho_f & 0 & \rho_c & 0 \\ 0 & \rho_f & 0 & \rho_c \end{pmatrix},\quad Q_2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \eta/\kappa & 0 \\ 0 & 0 & 0 & \eta/\kappa \end{pmatrix},$$

respectively. In D_1,

$$\alpha=1-\frac{K_b}{K_s},\quad M=\frac{K_s}{\alpha+\beta\big(\frac{K_s}{K_f}-1\big)},\quad \lambda_c=\lambda_b+\alpha^2 M,$$

λ_b and μ_b are the Lame coefficients of the porous solid; K_b, K_s and K_f the bulk modulus of the porous solid, the nonporous solid (i.e., solid material) and the fluid, respectively, $K_b=\lambda_b+(2/3)\mu_b$. In Q_1, ρ is the density of the solid-fluid aggregate,

$$\rho+\rho_1+\rho_2=(1-\beta)\rho_s+\beta\rho_f,$$

ρ_s and ρ_f are the densities of the nonporous solid and the fluid, respectively. When the aggregate moves, the mass coupling effect occurs. Hence ρ_1 and ρ_2 should be corrected into

$$\rho_{11}=\rho_1-\rho_{12},\quad \rho_{22}=\rho_2-\rho_{12},$$

where ρ_{12} is a mass coupling coefficient. $\rho_c=\rho_{22}/\beta^2$. In Q_2, η is the viscosity, κ the permeability.

The equations in Ω_2 are

$$B_2^T\Big(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\Big)D_2B_2\Big(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\Big)u=\rho\frac{\partial^2 u}{\partial t^2}+F_2,\quad (x,y,t)\in\Omega_2\times(0,T), \tag{2.2}$$

where

$$B_2\Big(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\Big)=\begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix},\quad D_2=\begin{pmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{pmatrix},$$

λ and μ are the Lame coefficients of the elastic solid in Ω_2.

The wave source energy should be partitioned between the solid and the fluid phases in the following proportion

$$P_s=1-\beta,\quad P_f=\beta,$$

where P_s and P_f are the weighting factors for the solid and the fluid motions, respectively. Because the motion of the fluid relative to the solid is considered, P_f should be replaced by the following factor

$$\overline{P}_f = \beta(P_f - P_s) = \beta(2\beta - 1).$$

Thus the right-hand side F_1 of (2.1) has the following form

$$F_1 = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} (1-\beta)\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \\ \beta(2\beta-1)\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \end{pmatrix}.$$

F_2 in (2.2) is a two-dimensional vector as usual.

The boundary conditions are as follows

$$B_1^T(n_x, n_y) D_1 B_1\Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) W = G_1, \ \ (x, y, t) \in \Gamma_{1s} \times (0, T], \tag{2.3}$$

$$B_2^T(n_x, n_y) D_2 B_2\Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) u = G_2, \ \ (x, y, t) \in \Gamma_{2s} \times (0, T], \tag{2.4}$$

$$\mathcal{B}_1(x, y, t; W) = 0, \ \ (x, y, t) \in \Gamma_{1a} \times (0, T], \tag{2.5}$$

$$\mathcal{B}_2(x, y, t; u) = 0, \ \ (x, y, t) \in \Gamma_{2a} \times (0, T]. \tag{2.6}$$

Here $n = (n_x, n_y)$ is the unit outward normal vector along the boundary $\partial\Omega$; $G_1 = \{(1-\beta) f_{1n}, f_{1s}, \beta(2\beta-1) f_{1n}, 0\}^T$ and $G_2 = \{f_{2n}, f_{2s}\}^T$, f_{in} and f_{is} $(i = 1, 2)$ are the normal and tangential forces on the boundaries Γ_{is}, which are known; $\mathcal{B}_1 = 0$ and $\mathcal{B}_2 = 0$ are ABCs, which will be discussed in the next section.

The connective conditions on the interface Γ_{12} between two different media are

$$\begin{cases} u_x^{(1)} = u_x^{(2)}, \ u_y^{(1)} = u_y^{(2)}, \ w_x^{(1)} n_x + w_y^{(1)} n_y = 0, \\ \tau_{nn}^{(1)} = \sigma_{nn}^{(2)}, \ \tau_{ss}^{(1)} = \sigma_{ss}^{(2)}, \ p_f^{(1)} = \sigma_{nn}^{(2)}, \end{cases} \quad (x, y, t) \in \Gamma_{12} \times (0, T], \tag{2.7}$$

where the upper index 1 means in Ω_1, and the upper index 2 means in Ω_2,

$$\tau_{nn}^{(1)} = \sigma_{nn}^{(1)} - \beta p_f n_x, \quad \tau_{ss}^{(1)} = \sigma_{ss}^{(1)} - \beta p_f n_y,$$

$\sigma_{nn}^{(i)}$ and $\sigma_{ss}^{(i)}$ $(i = 1, 2)$ are the normal and tangential stress components of the elastic solid in Ω_1 and Ω_2, respectively, and p_f is the pressure of the fluid in the solid-fluid aggregate.

The initial conditions are

$$\begin{aligned} &u(x, y, 0) = u_0, \ \ \frac{\partial u}{\partial t}(x, y, 0) = u_1, \ \ (x, y) \in \Omega, \\ &w(x, y, 0) = w_0, \ \ \frac{\partial w}{\partial t}(x, y, 0) = w_1, \ \ (x, y) \in \Omega_1, \end{aligned} \tag{2.8}$$

where u_0, u_1, w_0 and w_1 are konwn vectors.

3. Absorbing Boundary Conditions

In this section, a kind of discrete ABCs developed by the author of this paper and her cooperator will be presented. It has uniform formulas for the wave equations of elastic solids and fluid-saturated porous media. Thus it is convenient to be used for the problems discussed in this paper.

It is well known that the stability of computation is a serious problem when ABCs of high orders are used, and the higher the order of ABCs, the better the accuracy of solutions, but the worse the stability of computation. Taking into account of both accuracy and stability, only the second order ABCs will be considered in this section.

The basic idea of our ABCs is as follows. As is well known, in isotropic elastic media, there are two waves propagating: dilatational and rotational waves with velocities C_p and C_s, respectively, and in fluid-saturated porous media, there are two dilatational waves with the velocities C_{p1} and C_{p2} and one rotational wave with the velocity C_s. While every wave mentioned above meets a boundary, reflected waves occur. When we look for the solutions of wave equations for certain theoretical or practical purposes, we often want to know how every incident or reflected wave propagates in considered domain. But the ABCs should eliminate all artificial refections. In this case, individual reflected wave is not of interest to us. Hence, we can only consider the composition of all reflected waves. We may assume that this composition wave behaves like an acoustic wave. Imagine that its velocity is C_A. C_A will be determined later. In such a way, we can obtain the ABCs by utilizing ABCs for the acoustic wave equation.

Without loss of generality, suppose that the artificial boundary is $x = a, a > 0$. One of discrete ABCs is the following [3, 5, 6]

$$V(a, y, t + \Delta t) = 2V(a - C_A\Delta t, y, t) - V(a - 2C_A\Delta t, y, t - \Delta t), \tag{3.1}$$

where V can be the vector u or W mentioned in Sec. 1. Generally speaking, the points $(a - C_A\Delta t, y)$ and $(a - 2C_A\Delta t, y)$ are not mesh points, and an interpolation is necessary. It can be proved that the following approach of interpolation can yield the most stable ABC: Increase two columns of mesh points with abscissae $x = a + \Delta x$ and $x = a + \Delta x + 2C_A\Delta t$, respectively, and take $x = a + \Delta x + 2C_A\Delta t$ as a new artificial boundary on which the ABC (3.1) is used. It is

$$V(x + \Delta x + 2C_A\Delta t, y, t + \Delta t) = 2V(a + \Delta x + C_A\Delta t, y, t) - V(a + \Delta x, y, t - \Delta t). \tag{3.2}$$

By using the Lagrange interpolation formula, evaluate $V(a + \Delta x + C_A\Delta t, y, t)$ from the values of V at the points $(a + \Delta x + 2C_A\Delta t, y, t), (a + \Delta x, y, t)$ and (a, y, t). Then the following boundary condition can be obtained

$$\begin{aligned} V(a + \Delta x + 2C_A\Delta t, y, t + \Delta t) &= \alpha_1 V(a + \Delta x + 2C_A\Delta t, y, t) \\ &+ \alpha_2 V(a + \Delta x, y, t) + \alpha_3 V(a, y, t) - V(a + \Delta x, y, t - \Delta t), \end{aligned} \tag{3.3}$$

where

$$\alpha_1 = \frac{1+R}{1+2R}, \quad \alpha_2 = 1+R, \quad \alpha_3 = -\frac{2R^2}{1+2R}, \quad R = \frac{C_A \Delta t}{\Delta x}. \tag{3.4}$$

Remark 1. We mentioned the terms "the most stable ABC" before. Here it means that α_1 is the minimum.

We determine the velocity C_A of the composed reflected wave by the way to make the reflection coefficients minimal. By reflection coefficient we mean the ratio of the amplitude of reflected wave to the amplitude of incident wave. On the basis of this idea, it can be proved (see [4]) that for the isotropic elastic media, the formula to find C_A is

$$\max_{i=1,2} \left| \frac{C_A(x,y)}{C_i(x,y)} - 1 \right| = \min_{C_2(x,y) \le C \le C_1(x,y)} \max_{i=1,2} \left| \frac{C}{C_i(x,y)} - 1 \right|$$

for every $(x,y) \in \Gamma_{2a}$, where

$$C_1 = C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad C_2 = C_s = \sqrt{\frac{\mu}{\rho}},$$

and for the fluid-saturated porous media, it is

$$\max_{i=1,2,3} \left| \frac{C_A(x,y)}{C_i(x,y)} - 1 \right| = \min_{C_{\min}(x,y) \le C \le C_{\max}(x,y)} \max_{i=1,2,3} \left| \frac{C}{C_i(x,y)} - 1 \right|$$

for every $(x,y) \in \Gamma_{1a}$, where

$$C_{1,2} = \left\{ \frac{\rho_c H + \rho M - 2\rho_f \alpha M \pm \left[(\rho_c H + \rho M - 2\rho_f \alpha M)^2 - 4(\rho\rho_c - \rho_f^2)(HM - \alpha^2 M^2) \right]^{1/2}}{2(\rho\rho_c - \rho_f^2)} \right\}^{\frac{1}{2}}$$

$$C_3 = \left\{ \frac{\rho_c \mu_b}{\rho\rho_c - \rho_f^2} \right\}^{\frac{1}{2}},$$

$$H = \lambda_b + 2\mu_b + \alpha^2 M,$$

$$C_{\max}(x,y) = C_1(x,y), \quad C_{\min}(x,y) = \min\{C_2(x,y), C_3(x,y)\}.$$

4. The Discrete-Time Galerkin Method

Subdivide the domain into triangle or quadrilateral elements. Finite element methods of triangle elements with linear basis functions or isoparametric quadrilateral elements with bilinear basis functions can be applied. In order to have finite element space in which the connective conditions (2.7) on the interface Γ_{12} can be easily imposed, the subdivision should be compatible in the sense that if an edge of a triangle or quadrilateral element in Ω_1 is contained in Γ_{12}, then this edge is also an edge of a triangle or quadrilateral element in Ω_2. Suppose that the number of nodal points is N. Denote the nodal points as $P_1, \dots, P_N$. At the point P_i, the global basis function $\varphi_i(x,y)$ can be obtained from the element basis functions. It possesses the feature $\varphi_i(P_j) = \delta_{ij}$. Introduce the diagonal matrix $\Phi_i^{(1)}(x,y) = \operatorname{diag}(\varphi_i(x,y), \varphi_i(x,y), \varphi_i(x,y), \varphi_i(x,y))$ and the diagonal matrix $\Phi_i^{(2)}(x,y) = \operatorname{diag}(\varphi_i(x,y), \varphi_i(x,y))$ and expand the displacement vectors W and u in the following forms

$$W(x,y,t) = \sum_{i=1}^{N} \Phi_i^{(1)}(x,y) W_i(t), \quad (x,y) \in \Omega_1,$$
$$u(x,y,t) = \sum_{i=1}^{N} \Phi_i^{(2)}(x,y) u_i(t), \quad (x,y) \in \Omega_2,$$

where W_i and u_i are the values of W and u at the point P_i, respectively.

The continuous-time Galerkin equations of the problem (2.1) - (2.8) is the following

$$\begin{aligned}
&\frac{\partial^2}{\partial t^2} \iint_{\Omega_1} \Phi_i^{(1)} Q_1 W dxdy + \frac{\partial^2}{\partial t^2} \iint_{\Omega_2} \rho \Phi_i^{(2)} u dxdy + \frac{\partial}{\partial t} \iint_{\Omega_1} \Phi_i^{(1)} Q_2 W dxdy \\
&\quad + \iint_{\Omega_1} \Big\{ B_1\Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) \Phi_i^{(1)} \Big\}^T D_1 B_1 \Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) W dxdy \\
&\quad + \iint_{\Omega_2} \Big\{ B_2\Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) \Phi_i^{(2)} \Big\}^T D_2 B_2 \Big(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\Big) u dxdy \\
&= -\iint_{\Omega_1} \Phi_i^{(1)} F_1 dxdy - \iint_{\Omega_2} \Phi_i^{(2)} F_2 dxdy - \int_{\Gamma_{1s}} \Phi_i^{(1)} G_1 ds - \int_{\Gamma_{2s}} \Phi_i^{(2)} G_2 ds, \\
&\quad i = 1, \dots, N. \qquad (4.1)
\end{aligned}$$

It is a system of ODEs of the following form

$$M_1 \ddot{W} + M_2 \ddot{u} + C \dot{W} + S_1 W + S_2 u = F, \qquad (4.2)$$

where M_1 and M_2 are mass matrices, C damping matrix, S_1 and S_2 stiffness matrices.

Solve the system (4.2) by employing finite difference method. Let the time step size is Δt. Take the following scheme

$$\frac{W^{n+1} - W^n}{\Delta t} = V^{n+1},$$
$$\frac{u^{n+1} - u^n}{\Delta t} = v^{n+1},$$
$$M_1 \frac{V^{n+1} - V^n}{\Delta t} + M_2 \frac{v^{n+1} - v^n}{\Delta t} + CV^n + S_1 W^n + S_2 u^n = F^n,$$

that is

$$\begin{aligned}
&M_1 V^{n+1} + M_2 v^{n+1} = M_1 V^n + M_2 v^n - \Delta t C V^n - \Delta t S_1 W^n - \Delta t S_2 u^n + \Delta t F^n, \\
&W^{n+1} = W^n + \Delta t V^{n+1}, \qquad (4.3) \\
&u^{n+1} = u^n + \Delta t v^{n+1}.
\end{aligned}$$

If P_i is in Ω_1, the elements of the matrix M_2 corresponding to P_i are zero, and similarly, if P_i is in Ω_2, the corresponding elements of matrix M_1 are zero. After a permutation of unknowns, M_1 can be reduced to a 2×2 block diagonal matrix, and through a lumping process, M_2 can be replaced by a diagonal matrix. Consequently, (4.3) can easily be rewritten as an explicit scheme.

Remark 2. 1) At every time step, the absorbing boundary condition (3.3) is a Dirichlet boundary condition, i.e., on the nodal points of Γ_{1a} and Γ_{2a}, the function value are given. Thus the integrals on Γ_{1a} and Γ_{2a} disappears in the equation (4.1).
2) The integrals on Γ_{12} also disappears in (4.1) because of the connective conditions (2.7).

5. Numerical Examples

Example 1. $\Omega = \Omega_1 = \big\{(x,y) = -1500 \le x \le 1500,\ 0 \le y \le 1500\big\}$, $\Gamma_{1s} = \big\{(x,y) : -1500 \le x \le 1500,\ \ y = 0\big\}$, $\Gamma_{1a} = \partial\Omega/\Gamma_{2s}$. An explosive source is set at the point (0, 400) (because of the limited space of the paper, we will not write its expression). The medium parameters are

$$\lambda_b = 5.3568 \times 10^6, \quad \mu_b = 4.32 \times 10^6, \quad K_f = 2.25 \times 10^6,$$
$$K_s = 10.296 \times 10^6, \quad \beta = 0.2, \quad \kappa = 0.4,$$
$$\eta = 1.0 \times 10^{-12}, \quad \rho_f = 1.0, \quad \rho_s = 2.4, \quad \rho_{12} = -0.4.$$

The numerical results are shown in Fig. 1. In this case, only the dilatational waves P_1 and P_2 are generated and visible in the figure.

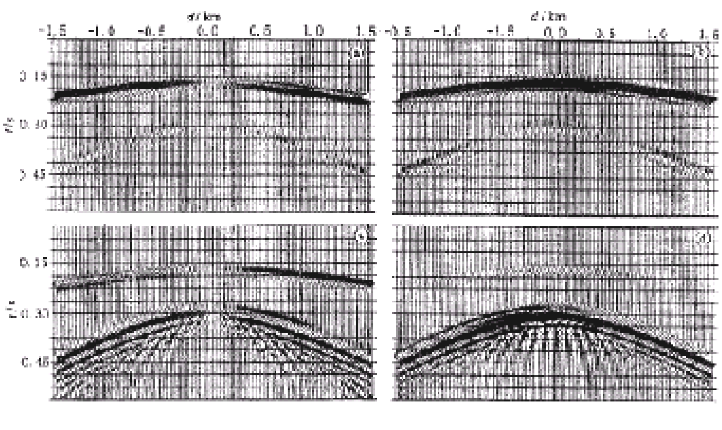

Fig. 1. Synthetic seismogram receivers are located on $y = 0$
(a) and (b) are the horizontal and vertical components u_x and u_y of the solid displacement,
(c) and (d) are the horizontal and vertical components w_x and w_y of the displacement of the fluid relative to the solid.

Example 2. The domain and the medium parameters are the same as in the Example 1. A perpendicular concentrative force acts at the point (0, 0).

The numerical results are shown in Fig. 2. In this case, the dilatational wave P_1 and P_2 and the rotational wave S all appear, but the P_2-wave is weak.

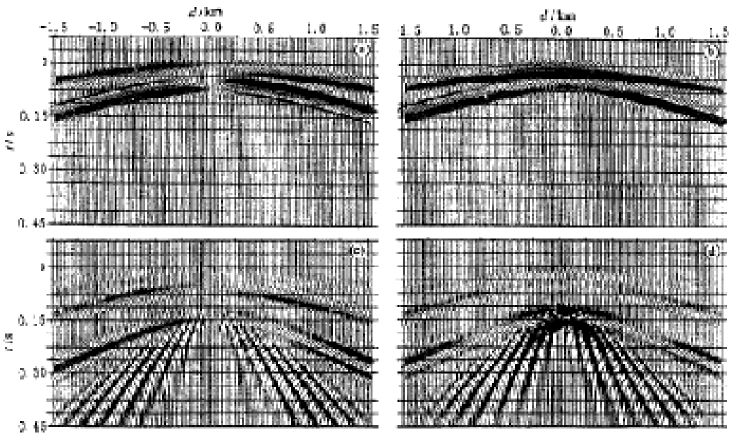

Fig. 2. Synthetic seismogram receivers are located on $y = 0$
(a) and (b) are the horizontal and vertical components u_x and u_y of the solid displacement, (c) and (d) are the horizontal and vertical components w_x and w_y of the displacement of the fluid relative to the solid.

In Fig. 1 and Fig. 2, it can be seen that a dispersion of P_2-wave occurs in the displacement of the fluid relative to the solid. Some authors have noticed and discussed this phenomenon.

The results of numerical experiments have shown that the ABCs and the numerical method suggested in this paper are effective.

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