A Remark on the Dirichlet Problem*

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Abstract. Given a positive measure $\mu$ on a strongly pseudoconvex domain in $\mathbb{C}^n$. We study the Dirichlet problem $(dd^cu)^n = \mu$ in a new class of plurisubharmonic function. This class includes the classes $E_p$ ($p \geq 1$) introduced by Cegrell in [5].

1. Introduction.

Let $\Omega$ be a bounded domain in $\mathbb{C}^n$. By $PSH(\Omega)$ we denote the set of plurisubharmonic (psh) functions on $\Omega$. By the fundamental work of Bedford and Taylor [1, 2], the complex Monge-Ampere operator $(dd^cu)^n$ is well defined over the class $PSH(\Omega) \cap L^\infty_{loc}(\Omega)$ of locally bounded psh functions on $\Omega$, more precisely, if $u \in PSH(\Omega) \cap L^\infty_{loc}(\Omega)$ is a positive Borel measure. Furthermore, this operator is continuous with respect to increasing and decreasing sequences. Later, Demailly has extended the domain of definition of the operator $(dd^cu)^n$ to the class of psh functions which are locally bounded near $\partial \Omega$. Recently in [5, 6], Cegrell introduced the largest class of upper bounded psh functions on a bounded hyperconvex domain $\Omega$ such that the operator $(dd^cu)^n$ can be defined on it. In these papers, he also studied the Dirichlet problems for the classes $F_p$ (see Sec. 2 for details). The aim of our work is to investigate the Dirichlet problem for a new class of psh function. This class consist, in particular, the sum of a function in the class $E_p$ and a function in $B^c_{loc}$ (see Sec. 2 for the definitions of these classes).

Now we are able to formulate the main result of our work

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Main theorem.
(i) Let $\Omega$ be a bounded strongly pseudoconvex domain in $\mathbb{C}^n$ and let $\mu$ be a positive measure on $\Omega$, $h \in C(\partial \Omega)$ such that there exists $v \in \mathcal{E}_p + \mathcal{B}_{lo}^a$ (resp. $\mathcal{F}_p + \mathcal{B}_{lo}^a$) with $(dd^c v)^n \geq \mu$
Then there exists $u \in \mathcal{E}_p + \mathcal{B}_{lo}^a$ (resp. $\mathcal{F}_p + \mathcal{B}_{lo}^a$) such that $(dd^c u)^n = \mu$ and $\lim_{z \to \xi} u(z) = h(\xi), \forall \xi \in \partial \Omega$.
(ii) There exists $f \in L^1(\Omega)$ such that there exists no function $u \in \mathcal{E}_p + \mathcal{B}_{loc}$ which satisfying $fd\lambda \leq (dd^c u)^n$.

For the definitions of $\mathcal{E}_p + \mathcal{B}_{loc}$ and $\mathcal{F}_p + \mathcal{B}_{loc}$ see Sec. 2.

Note that the main theorem for the subclass $\mathcal{B}$ of $\mathcal{B}_{loc}$ consisting of psh functions which are bounded near $\partial \Omega$ was proved by Xing in [13] and for the classes $\mathcal{E}_p$ and $\mathcal{F}_p$, $p \geq 1$ by Cegrell in [5].

The key element in the proof of our main theorem is a comparison principle (Theorem 3.1), which is an extension of Lemma 4.4, Theorem 4.5 in [5].

2. Preliminaries

In this section we recall some elements and results of pluripotential theory that will be used throughout the paper. All this can be found in [2, 3, 5, 6, 11...].

2.0. Unless otherwise specified, $\Omega$ will be a bounded hyperconvex domain in $\mathbb{C}^n$ meaning that there exists a negative exhaustive psh function for $\Omega$.

2.1. Let $\Omega$ be a bounded domain in $\mathbb{C}^n$. The $C_n$-capacity in the sense of Bedford and Taylor on $\Omega$ is the set function given by

$$C_n(E) = C_n(E, \Omega) = \sup \left\{ \int_E (dd^c u)^n : u \in PSH(\Omega), -1 \leq u \leq 0 \right\}$$

for every Borel set $E$ in $\Omega$.

2.2. According to Xing (see [13]), a sequence of positive measures $\{\mu_j\}$ on $\Omega$ is called uniformly absolutely continuous with respect to $C_n$ in a subset $E$ of $\Omega$ if

$$\forall \epsilon > 0, \exists \delta > 0 : F \subset E, C_n(F) < \delta \Rightarrow \mu_j(F) < \epsilon, \forall j \geq 1$$

We write $\mu_j \ll C_n$ in $E$ uniformly for $j \geq 1$.

2.3. By $\mathcal{B}_{loc}^a = \mathcal{B}_{loc}^a(\Omega)$ we denote the set of upper bounded psh functions $u$ which are locally bounded near $\partial \Omega$ such that $(dd^c u)^n \ll C_n$ in every $E \subset \subset \Omega$.

2.4. The following classes of psh functions were introduced by Cegrell in [5] and [6]

$$\mathcal{E}_0 = \mathcal{E}_0(\Omega) = \left\{ \varphi \in PSH(\Omega) \cap L^\infty(\Omega) : \lim_{z \to \partial \Omega} \varphi(z) = 0, \int_{\Omega} (dd^c \varphi)^n < +\infty \right\},$$

$$\mathcal{E}_p = \mathcal{E}_p(\Omega) = \left\{ \varphi \in PSH(\Omega) : \exists \mathcal{E}_0 \ni \varphi_j \searrow \varphi, \sup_{j \geq 1} \int_{\Omega} (-\varphi_j)^p (dd^c \varphi_j)^n < +\infty \right\},$$
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\[ \mathcal{F}_p = \mathcal{F}_p(\Omega) = \left\{ \varphi \in PSH(\Omega) : \exists \mathcal{E}_0 \ni \varphi_j \searrow \varphi, \sup_{j \geq 1} \int_{\Omega} (-\varphi_j)^p (dd^c \varphi_j)^n, \right. \\
\left. < + \infty, \sup_{j \geq 1} \int_{\Omega} (dd^c \varphi_j)^n < +\infty \right\} \]

\[ \mathcal{E} = \mathcal{E}(\Omega) = \left\{ \varphi \in PSH(\Omega) : \forall z_0 \in \Omega \exists \text{ a neighborhood } \omega \ni \varphi_j \searrow \varphi \text{ on } \omega, \sup_{j \geq 1} \int_{\Omega} (dd^c \varphi_j)^n < +\infty \right\}. \]

The following inclusions are obvious \( \mathcal{E}_0 \subset \mathcal{F}_p \subset \mathcal{E} \). It is also known that these inclusion are strict (see [5, 6]).

The interesting theorem below was proved by Cegrell (see [6])

**Theorem 2.5.** The class \( \mathcal{E} \) has the following properties
1. \( \mathcal{E} \) is a convex cone.
2. If \( u \in \mathcal{E}, v \in PSH^{-}(\Omega) = \{ \varphi \in PSH(\Omega) : \varphi \leq 0 \}, \) then \( \max(u, v) \in \mathcal{E} \).
3. If \( u \in \mathcal{E}, PSH(\Omega) \cap L^\infty_{\text{loc}}(\Omega) \ni u_j \searrow u, \) then \( (dd^c u_j)^n \) is weakly convergent.

Conversely if \( K \subset PSH^{-}(\Omega) \) satisfies 2 and 3, then \( K \subset \mathcal{E} \).

Since \( B^-_{\text{loc}} = B^a_{\text{loc}} \cap PSH^{-}(\Omega) \) satisfies 2 and 3 we have by [8] \( B^-_{\text{loc}} \subset \mathcal{E} \).

2.6. Cegrell also studied the following Dirichlet problem: Given a positive measure on \( \Omega \), find \( u \in \mathcal{F}_p \) such that \( (dd^c u)^n = \mu \). He gave a necessary and sufficient condition for this problem to have a solution (Theorem 5.2 in [5]).

2.7. We define

\[ \mathcal{E}_p + B^a_{\text{loc}} = \{ u \in PSH(\Omega) : \exists \varphi \in \mathcal{E}_p, f \in B^a_{\text{loc}} : \varphi + f \leq u \leq \sup_{\Omega} u < +\infty \}, \]

\[ \mathcal{F}_p + B^a_{\text{loc}} = \{ u \in PSH(\Omega) : \exists \varphi \in \mathcal{F}_p, f \in B^a_{\text{loc}} : \varphi + f \leq u \leq \sup_{\Omega} u < +\infty \}. \]

It follows that if \( \varphi + f \leq u < \sup_{\Omega} u < +\infty, \varphi \in \mathcal{E}, f \in B^a_{\text{loc}} \) then

\[ u - c = \max_{\Omega} (u - c, \varphi + f - c) \in \mathcal{E}, \text{ because } \varphi + (f - c) \in \mathcal{E}, \]

where \( c = \max_{\Omega} (sup f, sup u) \).

Thus we can define \( (dd^c u^n) \) for \( u \in \mathcal{E}_p + B^a_{\text{loc}} \).

2.8. The aim of this work is to study a Dirichlet problem similar to the one considered by Cegrell but for the classes \( \mathcal{E}_p + B^a_{\text{loc}} \) and \( \mathcal{F}_p + B^a_{\text{loc}} \). Namely, given a positive measure \( \mu \) on \( \Omega \) and \( h \in C(\partial \Omega) \), find \( u \in \mathcal{E}_p + B^a_{\text{loc}} \) (resp. \( \mathcal{F}_p + B^a_{\text{loc}} \)) such that \( (dd^c u)^n = \mu \) and \( \lim_{z \rightarrow \xi} u(z) = h(\xi) \forall \xi \in \partial \Omega \).

2.9. Let \( \mu \) be a positive measure on \( \Omega \) and \( h \in C(\partial \Omega) \). Following Cegrell, we define
\[ B(\mu, h) = \{ v \in PSH(\Omega) \cap L^\infty_{\text{loc}}(\Omega) : (dd^c v)^n \geq \mu, \lim_{z \to \xi} v(z) \leq h(\xi) \}, \]

\[ U(\mu, h)(z) = \sup \{ v(z) : v \in B(\mu, h) \}, \quad z \in \Omega. \]

Observe that \( B(\mu, h) \neq \emptyset \) implies that \( \mu \) vanishes on pluripolar sets. The function \( U(\mu, h) \) plays a crucial role in solving the Dirichlet problem.

### 3. The Comparison Principle for \( \mathcal{E}_p + \mathcal{B}_{\text{loc}}^0 \)

In order to prove the main theorem, in this section we prove the following comparison principle

**Theorem 3.1.** Let \( u, v \) be functions in \( \mathcal{E}_p + \mathcal{B}_{\text{loc}}^0 \) satisfying

\[ \lim_{z \to \partial \Omega} [u(z) - v(z)] \geq 0. \]

Then

\[ \int_{\{u < v\}} (dd^c v)^n \leq \int_{\{u < v\}} (dd^c u)^n. \]

We need the following result

**Lemma 3.2.** Let \( PSH(\Omega) \cap L^\infty(\Omega) \ni u_j \searrow u \). Assume that

\[ \lim_{s \to +\infty} s^n C_n(\{u < -s\} \cap D) = 0, \quad \forall D \subset \subset \Omega. \]

Then \( (dd^c u_j)^n \ll C_n \) in every \( D \subset \subset \Omega \) uniformly for \( j \geq 1 \).

**Proof.** Given \( D \subset \subset \Omega \). Without loss of generality we may assume that \( D \) is hyperconvex and \( u_j \leq 0 \) on \( D \). By [6] for each \( j \geq 1 \) there exists \( u_j^k \in PSH(D) \cap C(\overline{D}) \) such that \( u_j^k \searrow u_j \) on \( D \) and \( u_j^k = 0 \) on \( \partial D \). As in [9] for every \( k, j \geq 1 \) and \( s > 0 \) put

\[ D_{kj}(s) = \{ u_j^k < -s \} \cap D, \quad D_j(s) = \{ u_j < -s \} \cap D, \quad D(s) = \{ u < -s \} \cap D, \]

\[ a_{kj}(s) = C_n(D_{kj}(s)), \quad a_j(s) = C_n(D_j(s)), \quad a(s) = C_n(D(s)), \]

\[ b_{kj}(s) = \int_{D_{kj}(s)} (dd^c u_j^k)^n, \quad b_j(s) = \int_{D_j(s)} (dd^c u_j)^n. \]

For \( 0 < s < t \) we have \( \max(u_j^k, -t) = u_j^k \) on \( \{ u_j^k > -t \} \) an open neighborhood of \( \partial D_{kj}(s) \). It follows that

\[ a_{kj}(s) \geq t^{-n} \int_{D_{kj}(s)} (dd^c \max(u_j^k, -t))^n = t^{-n} \int_{D_{kj}(s)} (dd^c u_j^k)^n = t^{-n} b_{kj}(s). \]
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Letting \( t \searrow s \) we get

\[
s^n a_{kj}(s) \geq b_{kj}(s) \quad \text{for } k, j \geq 1 \text{ and } s > 0. \tag{1}
\]

Given \( \epsilon > 0 \). By the hypothesis there exists \( s_0 > 0 \) such that

\[
s_0^n a(s_0) < \epsilon. \tag{2}
\]

Let \( E \subset D \) with \( C_n(E) < \frac{\delta}{s_0} \). Take an open neighborhood \( G \) of \( E \) such that \( C_n(G) < \frac{\delta}{s_0} \). Since \( (dd^c u_j^n) \to (dd^c u^n) \) weakly as \( k \to \infty \) we have

\[
\int_E (dd^c u_j^n) \leq \int_G (dd^c u_j^n) \leq \lim_{k \to \infty} \int_G (dd^c u_j^n) \]

\[
\leq \lim_{k \to \infty} \left( \int_{D_{kj}(s_0)} (dd^c u_j^n) + \int_{G \setminus D_{kj}(s_0)} (dd^c u_j^n) \right) \]

\[
< \lim_{k \to \infty} \left[ s_0^n a_{kj}(s_0) + s_0^n C_n(G) \right] \leq s_0^n a(s_0) + \epsilon < 2\epsilon
\]

for \( j \geq 1 \). Hence \( (dd^c u_j^n) \ll C_n \) in \( D \) uniformly for \( j \geq 1 \).

Proof of Theorem 3.1. We may assume that \( u, v \leq 0 \) and \( \lim_{z \to \partial \Omega} [u(z) - v(z)] > \delta > 0 \). By hypothesis \( u, v \in \mathcal{E}_p + \mathcal{B}_n^{\text{loc}} \) it is easy to find \( \varphi \in \mathcal{E}_p, g \in \mathcal{B}_n^{\text{loc}} \) such that \( \varphi + g \leq \min(u, v) \). Let \( \varphi_j \searrow \varphi \) be a sequence decreasing to \( \varphi \) as in the definition of \( \mathcal{E}_p \). For each \( j \geq 1 \) put

\[
g_j = \max(g, -j), \quad u_j = \max(u, \varphi_j + g_j), \quad v_j = \max(v, \varphi_j + g_j).
\]

It follows that \( g_j, u_j, v_j \) are bounded and \( g_j \searrow g, u_j \searrow u, v_j \searrow v \). By the comparison principle for bounded psh functions we have

\[
\int_{\{u_j < v_k\}} (dd^c v_k)^n \leq \int_{\{u_j < v_k\}} (dd^c u_j^n)
\]

for \( k \geq j \geq 1 \).

On the other hand, since

\[
s^n C_n(\{u < -s\} \cap D) \leq s^n C_n(\{\varphi < -\frac{s}{2}\} \cap D) + s^n C_n(\{g < -\frac{s}{2}\} \cap D) \to 0
\]

as \( s \to +\infty \) (see [5, 9])

By Lemma 3.2 \( (dd^c u_j^n) + (dd^c v_j)^n \ll C_n \) in every \( D \subset \subset \Omega \) uniformly for \( j \geq 1 \). Thus by the quasicontinuity of psh functions as Theorem 2.2.6 in [4] we obtain

\[
\int_{\{u \leq v\}} (dd^c v)^n \leq \int_{\{u \leq v\}} (dd^c u)^n.
\]

By replacing \( u \) by \( u + \delta \), \( \delta > 0 \) and then let \( \delta \searrow 0 \), we have
\[
\int_{\{u < v\}} (dd^c v)^n \leq \int_{\{u < v\}} (dd^c u)^n.
\]
This is the desired conclusion.

From Theorem 3.1, as Corollary 2.2.8 in [4], we get the following dominant principle.

**Corollary 3.3.** Assume that \( u \) and \( v \) are as in Theorem 3.1 and \((dd^c u)^n \leq (dd^c v)^n\). Then \( u \geq v \).

**4. Proof of the Main Theorem**

(i) We can assume \( v \leq 0 \). Since \((dd^c v)^n\) vanishes on every pluripolar set in \( \Omega \), by Theorem 6.3 in [5] we can find \( \psi \in E_0 \) and \( 0 \leq f \in L_1^{loc}(dd^c \psi)^n \) such that \( \mu = f(dd^c \psi)^n \). Put \( \mu_k = \min(f,k)(dd^c \psi)^n \). Then \( \mu_k \leq (dd^c \psi)^n \).

By Theorem 2 in [13] there exists \( \omega_k \in E_0 \) such that \((dd^c \omega_k)^n = \mu_k\). The comparison principle implies that \( 0 \geq \omega_k \searrow \omega \geq v \). Hence \( \omega \in E_p + B^a_{loc} \) and \((dd^c \omega)^n = \mu\). We show that \( \lim_{z \to \xi} \omega(z) = 0 \) for \( \xi \in \partial \Omega \). Assume the contrary, then \( \lim_{z \to \xi} \omega(z) < -\epsilon \) for some \( \xi_0 \in \partial \Omega \), \( \epsilon > 0 \). Take \( \delta > 0 \) such that \( \omega(z) < -\epsilon \) for \( z \in B(\xi_0, \delta) \cap \Omega \). Let \( \tau \in C(\partial \Omega) \) such that \( \tau|_{B(\xi_0, \delta) \cap \partial \Omega} = \epsilon \), \( \text{supp} \tau \subset B(\xi_0, \delta) \cap \partial \Omega \). By [2] there exists \( \phi \in PSH(\Omega) \cap C(\Omega) \) such that \((dd^c \phi)^n = 0 \) and \( \phi|_{\partial \Omega} = \tau \). Since \( \lim_{z \to \xi} [\omega_k(z) - (\omega(z) + \phi(z))] \geq 0 \) for \( \xi \in \partial \Omega \) and \((dd^c \omega_k)^n = \mu_k \leq \mu = (dd^c \omega)^n \leq (dd^c (\omega + \phi))^n\), we have \( \omega_k \geq \omega + \phi \) on \( \Omega \) for \( k \geq 1 \). Thus \( \omega \geq \omega + \phi \) on \( \Omega \). Hence \( \phi \leq 0 \) on \( \Omega \setminus \{\omega = -\infty\} \). Since \( \phi \) is plurisubharmonic, \( \phi \leq 0 \) on \( \Omega \). This is impossible, because \( \phi(\xi) = \tau(\xi) = \epsilon \) for \( \xi \in B(\xi_0, \delta) \cap \partial \Omega \). Hence \( \lim_{z \to \xi} \omega(z) = 0 \) for \( \xi \in \partial \Omega \). From the relations

\[
\begin{cases}
U((dd^c (\omega_k + U(0,h)))^n, h) = \omega_k + U(0,h), \\
(dd^c (\omega + U(0,h)))^n \geq \mu_k,
\end{cases}
\]

and from Theorem 8.1 in [5] it follows that

\[
\begin{cases}
(dd^c U(\mu_k, h))^n = \mu_k, \\
U(0,h) \geq U(\mu_k, h) \geq \omega_k + U(0,h).
\end{cases}
\]

Theorem 3.1 implies that \( U(\mu_k, h) \searrow u \in E_p + B^a_{loc} \) with \((dd^c u)^n = \mu\) and \( U(0,h) \geq u \geq \omega + U(0,h) \). Thus for \( \xi \in \partial \Omega \) we have

\[
h(\xi) = \lim_{z \to \xi} U(0,h) \geq \lim_{z \to \xi} u(z) \geq \lim_{z \to \xi} [\omega(z) + U(0,h)(z)]
= \lim_{z \to \xi} \omega(z) + \lim_{z \to \xi} U(0,h)(z) = h(\xi).
\]

Consequently \( u \in E_p + B^a_{loc} \) such that \((dd^c u)^n = \mu\) and \( \lim_{z \to \xi} u(z) = h(\xi) \forall \xi \in \partial \Omega \).
(ii) Let \( \{ \Omega_j \} \) be an increasing exhaustion sequence of strongly pseudoconvex subdomains of \( \Omega \). For each \( j \geq 1 \) take a sequence of distinguished points \( z_{jm} \subset \Omega_j \setminus \overline{\Omega}_{j-1} \) converging to \( \xi_j \in \partial \Omega_j \) as \( m \to \infty \) and a sequence \( s_j \searrow 0 \) such that \( B(z_{jm}, s_{jm}) \subset \Omega_j \setminus \overline{\Omega}_{j-1} \) and \( B(z_{jt}, s_{jt}) \cap B(z_{jt}, s_{jt}) = \emptyset \) for \( m \neq t \). Let \( a_{jm} > 0 \) with \( \sum_{j=1}^{\infty} a_{jm} < \infty \). Put
\[
 f = \sum_{j,m \geq 1} a_{jm} \chi_{B(z_{jm}, r_{jm})},
\]
where \( 0 < r_{jm} < s_{jm} \) are chosen such that
\[
 \frac{1}{a_{jm}} (C_n(B(z_{jm}, r_{jm}), \Omega))^{\frac{1}{n + p}} \to 0 \text{ as } m \to \infty,
\]
for \( j \geq 1 \) and \( d_n \) is the volume of the unit ball in \( \mathbb{C}^n \).

Assume that \( f \lambda \leq (\dd \bar{u}^n) \) for some \( u \in \mathcal{E}_p + B^0_{\mathcal{E} \cap \mathcal{E}_p} \). Take \( \varphi \in \mathcal{E}_p, \ g \in B^0_{\mathcal{E} \cap \mathcal{E}_p} \) such that \( \varphi + \varphi \leq u \leq \sup u + +\infty \). We may assume that \( g \) and \( u \) are negative.

Let \( j_0 \geq 2 \) and \( M > 0 \) such that \( g > -M \) on \( \Omega_{j_0} \setminus \overline{\Omega}_{j_0 - 1} \).

Put
\[
 \tilde{g} = \max(g, Ah_{\Omega_{j_0}}) \text{ where } A = -M \sup_{\Omega_{j_0}} h_{\Omega_{j_0}} > 0.
\]

It follows that \( \tilde{g} \in \mathcal{E}_0, \ \tilde{g} = g \) on \( \Omega_{j_0} \setminus \overline{\Omega}_{j_0 - 1} \).

Let \( \tilde{u} = \max(u, \varphi + \tilde{g}) \). Since \( \varphi + \tilde{g} \leq \tilde{u} \leq 0 \) and \( \varphi + \tilde{g} \in \mathcal{E}_p + \mathcal{E}_0 = \mathcal{E}_p \), by [5] we have \( \tilde{u} \in \mathcal{E}_p \).

Moreover \( \tilde{u} = u \) on \( \Omega_{j_0} \setminus \overline{\Omega}_{j_0 - 1} \). Thus for \( B_m = B(z_{jm}, r_{jm}) \) we have
\[
 a_{jm} = \int_{B_m} f d\lambda = \int_{B_m} (dd^c \bar{u})^n = \int_{B_m} (dd^c \tilde{u})^n.
\]

Let \( \tilde{u}_k \searrow \tilde{u} \) as in the definition of \( \mathcal{E}_p \). Then \( (dd^c \tilde{u}_k)^n \to (dd^c \tilde{u})^n \) weakly (see [5]). Applying the Holder inequality (see [7]) we have
\[
 a_{jm} = \int_{B_m} (dd^c \tilde{u})^n \leq \lim_{k \to \infty} \int_{B_m} (dd^c \tilde{u}_k)^n
 = \lim_{k \to \infty} \int_{B_m} (-h_{B_m})^p (dd^c \tilde{u}_k)^n
 \leq \alpha_1 \lim_{k \to \infty} \int_{\Omega} (-h_{B_m})^p (dd^c h_{B_m})^n)^{\frac{n}{n + p}} \int_{\Omega} (-\tilde{u}_k)^p (dd^c \tilde{u}_k)^n)^{\frac{n}{n + p}}
 \leq \alpha_2 \int_{\Omega} (dd^c h_{B_m})^n)^{\frac{n}{n + p}}
 = \alpha_2 |C_n(B_m, \Omega)|^{\frac{n}{n + p}},
\]
where \( \alpha_2 = \alpha_1 \sup_{k \geq 1} \int_{\Omega} (-\tilde{u}_k)^p (dd^c \tilde{u}_k)^n)^{\frac{n}{n + p}} < +\infty \). This is impossible, because
\[
\lim_{m \to \infty} \frac{[C_n(B_m, \Omega)]^{\frac{n}{p}}}{a_{jm}} = 0.
\]

Remark. Using Theorem 7.5 in [1] we can find \( u \in F^a \) such that \((dd^cu)^n = f d\lambda\) where \( f \) is constructed as in (ii). Hence, there exists a function \( u \) in \( F^a \setminus (\xi_p + B^a_{loc}) \).

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