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A Class of Alternating Group Explicit Finite Difference Method for Diffusion Equation

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Abstract. In this paper, we first present a group of asymmetric finite difference schemes to approach diffusion equation. Based on the schemes a class of alternating group explicit finite difference method is derived. The scheme is verified to be unconditionally stable, and can be used for parallel computation. Results of the numerical experiments show that the method is of high accuracy.

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1. Introduction

Recently many scientists payed much attention to the finite difference methods with the property of parallelism. Many parallel finite difference methods for the diffusion equations have been presented. We notice that a so-called AGE method is widely cared for its intrinsic parallelism and absolute stability. The AGE method is originally presented for solving diffusion equations in [2] by Evans, and is soon developed by many authors such as in [1, 5, 6, 7]. The developed methods have the same advantages as the AGE method in [2], that is, parallelism and absolute stability. But we notice that almost all the methods have no more than two order accuracy for spatial step in the case of using six grid points. In [3], a class of exponential type method is established for convection-diffusion equations, which has high accuracy even the diffusion coefficient is small. But the method is not suitable for diffusion equations.

In this paper, we will consider the following initial boundary value problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1,\ 0 \le t \le T \\ u(x,0) = f(x), \\ u(0,t) = g_1(t),\ u(1,t) = g_2(t). \end{cases} \tag{1}$$

The paper is organized as follows: In Sec. 2, we present an $O(\tau^2 + h^4)$ order implicit scheme for solving (1) at first. Based on the scheme we present four asymmetry schemes, and then construct a class of alternating group explicit method. In Sec. 3, stability and error analysis are given. In Sec. 4, results of several numerical examples are presented.

2. The Alternating Group Explicit Method

The domain $\Omega : (0,1) \times (0,T)$ will be divided into $(m \times N)$ meshes with spatial step size $h = \dfrac{1}{m}$ in x direction and the time step size $\tau = \dfrac{T}{N}$. Grid points are denoted by (x_i, t_n) or $(i,\ n)$, $x_i = ih$ $(i = 0, 1, \ldots, m)$, $t_n = n\tau$ $(n = 0, 1, \ldots, \frac{T}{\tau})$. The numerical solution of (1) is denoted by u_i^n, while the exact solution is $u(x_i,\ t_n)$. Let $r = \dfrac{a\tau}{h^2}$.

We present an implicit finite difference scheme with parameters for solving (1) as below:

$$\begin{aligned} &\xi_1 \frac{u_{i-1}^{n+1} - u_{i-1}^n}{\tau} + \xi_2 \frac{u_i^{n+1} - u_i^n}{\tau} + \xi_3 \frac{u_{i+1}^{n+1} - u_{i+1}^n}{\tau} \\ &= \frac{a}{2}\left[\frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{h^2} + \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{h^2})\right] \end{aligned} \tag{2}$$

Considering $\dfrac{\partial^k u}{\partial t^k} = a^k \dfrac{\partial^{2k} u}{\partial x^{2k}}$ and applying Taylor formula to (2) at (x_i, t_n), we have the truncation error:

$$\begin{aligned} &(\xi_1 + \xi_2 + \xi_3 - 1)a\frac{\partial^2 u}{\partial x^2} + (-\xi_1 + \xi_3)ah\frac{\partial^3 u}{\partial x^3} - \frac{1}{2}(\xi_1 + \xi_2 + \xi_3 - 1)a^2\tau\frac{\partial^4 u}{\partial x^4} \\ &+ \frac{1}{2}\left(\xi_1 + \xi_3 - \frac{1}{6}\right)ah^2\frac{\partial^4 u}{\partial x^4} + \frac{1}{6}(\xi_3 - \xi_1)ah^3\frac{\partial^5 u}{\partial x^5} + \frac{1}{2}(\xi_1 - \xi_3)a^2\tau h\frac{\partial^5 u}{\partial x^5} \\ &+ \frac{1}{4}\left(-\xi_1 - \xi_3 + \frac{1}{6}\right)a^2\tau h^2\frac{\partial^6 u}{\partial x^6} + \frac{1}{12}(\xi_1 - \xi_3 - \frac{1}{6})a^2\tau h^3\frac{\partial^7 u}{\partial x^7} + O(\tau^2 + h^4). \end{aligned}$$

Let

$$\begin{cases} \xi_1 + \xi_2 + \xi_3 - 1 = 0, \\ -\xi_1 + \xi_3 = 0, \\ \xi_1 + \xi_3 - \frac{1}{6} = 0, \end{cases}$$

that is $\xi_1 = \xi_3 = \dfrac{1}{12},\ \xi_2 = \dfrac{5}{6}$.

We denote (2) as

$$\begin{aligned} &(1-6r)u_{i+1}^{n+1} + (10+12r)u_i^{n+1} + (1-6r)u_{i+1}^{n+1} \\ &= (1+6r)u_{i-1}^{n} + (10-12r)u_i^{n} + (1+6r)u_{i+1}^{n}. \end{aligned} \tag{3}$$

And we can easily have that the truncation error of (3) is $O(\tau^2 + h^4)$.

Based on the scheme, we will present four asymmetry schemes:

$$\begin{aligned} &\frac{5}{6}\frac{u_i^{n+1} - u_i^n}{\tau} + \frac{1}{6}\frac{u_{i+1}^{n+1} - u_{i+1}^n}{\tau} \\ &= a\left(\frac{u_{i+1}^{n+1} - u_i^{n+1} - u_i^n + u_{i-1}^n}{2h^2} + \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{2h^2}\right), \end{aligned} \tag{4}$$

$$\begin{aligned} &\frac{1}{6}\frac{u_{i-1}^{n+1} - u_{i-1}^n}{\tau} + \frac{5}{6}\frac{u_i^{n+1} - u_i^n}{\tau} \\ &= a\left(\frac{u_{i+1}^{n+1} - u_i^{n+1} - u_i^n + u_{i-1}^n}{2h^2} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{2h^2}\right), \end{aligned} \tag{5}$$

$$\begin{aligned} &\frac{5}{6}\frac{u_i^{n+1} - u_i^n}{\tau} + \frac{1}{6}\frac{u_{i+1}^{n+1} - u_{i+1}^n}{\tau} \\ &= a\left(\frac{u_{i+1}^{n} - u_i^{n} - u_i^{n+1} + u_{i-1}^{n+1}}{2h^2} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{2h^2}\right), \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{1}{6}\frac{u_{i-1}^{n+1} - u_{i-1}^n}{\tau} + \frac{5}{6}\frac{u_i^{n+1} - u_i^n}{\tau} \\ &= a\left(\frac{u_{i+1}^{n} - u_i^{n} - u_i^{n+1} + u_{i-1}^{n+1}}{2h^2} + \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{2h^2}\right). \end{aligned} \tag{7}$$

We can rewrite (4)-(7) as below :

$$(5+3r)u_i^{n+1} + (1-3r)u_{i+1}^{n+1} = 6ru_{i-1}^n + (5-9r)u_i^n + (1+3r)u_{i+1}^n, \tag{8}$$

$$(1-3r)u_{i-1}^{n+1} + (5+9r)u_i^{n+1} - 6ru_{i+1}^{n+1} = (1+3r)u_{i-1}^n + (5-3r)u_i^n, \tag{9}$$

$$-6ru_{i-1}^{n+1} + (5+9r)u_i^{n+1} + (1-3r)u_{i+1}^{n+1} = (5-3r)u_i^n + (1+3r)u_{i+1}^n, \tag{10}$$

$$(1-3r)u_{i-1}^{n+1}+(5+3r)u_i^{n+1}=(1+3r)u_{i-1}^n+(5-9r)u_i^n+6ru_{i+1}^n. \qquad (11)$$

Based on (8)-(11) and (3), we construct three basic explicit computing point groups: (1) "$\kappa 1$" group: four grid points are involved, and (8), (3), (9), (10), (3), (11) are used respectively. Let $U_i^n=\left(u_i^n,u_{i+1}^n,u_{i+2}^n,u_{i+3}^n,u_{i+4}^n,u_{i+5}^n\right)^T$, then we have

$$A_1U_i^{n+1}=B_1U_i^n+F_i^n, \qquad (12)$$

here $F_i^n=\left(6ru_{i-1}^n,0,0,0,0,6ru_{i+1}^n\right)^T$,

$$A_1=\begin{pmatrix}5+3r & 1-3r & 0 & 0 & 0 & 0\\ 1-6r & 10+12r & 1-6r & 0 & 0 & 0\\ 0 & 1-3r & 5+9r & -6r & 0 & 0\\ 0 & 0 & -6r & 5+9r & 1-3r & 0\\ 0 & 0 & 0 & 1-6r & 10+12r & 1-6r\\ 0 & 0 & 0 & 0 & 1-3r & 5+3r\end{pmatrix},$$

$$B_1=\begin{pmatrix}5-9r & 1+3r & 0 & 0 & 0 & 0\\ 1+6r & 10-12r & 1+6r & 0 & 0 & 0\\ 0 & 1+3r & 5-3r & 0 & 0 & 0\\ 0 & 0 & 0 & 5-3r & 1+3r & 0\\ 0 & 0 & 0 & 1+6r & 10-12r & 1+6r\\ 0 & 0 & 0 & 0 & 1+3r & 5-9r\end{pmatrix}.$$

Then the numerical solution at grid nodes $(i,n+1)$, $(i+1,n+1)$, $(i+2,n+1)$, $(i+3,n+1)$ can be obtained explicitly as below:

$$U_i^{n+1}=A_1^{-1}(B_1U_i^n+F_i^n).$$

(2) "$\kappa 2$" group: three inner points are involved, and (8), (3), (9) are used respectively. Let $\overline{U}_i^n=(u_i^n,u_{i+1}^n,u_{i+2}^n)^T$, then we have

$$A_2\overline{U}_i^{n+1}=B_2\overline{U}_i^n+\overline{F}_i^n, \qquad (13)$$

here $\overline{F}_i^n=\left(6ru_{i-1}^n,0,6ru_{i+1}^{n+1}\right)^T$,

$$A_2=\begin{pmatrix}5+3r & 1-3r & 0\\ 1-6r & 10+12r & 1-6r\\ 0 & 1-3r & 5+9r\end{pmatrix},\quad B_2=\begin{pmatrix}5-9r & 1+3r & 0\\ 1+6r & 10-12r & 1+6r\\ 0 & 1+3r & 5-3r\end{pmatrix}.$$

The numerical solution at grid nodes $(i,n+1)$, $(i+1,n+1)$, $(i+2,n+1)$ can be denoted as below:

$$\overline{U}_i^{n+1}=A_2^{-1}(B_2\overline{U}_i^n+\overline{F}_i^n).$$

(3) "$\kappa 3$" group: three inner points are involved, and (10), (3), (11) are used respectively. Let $\widetilde{U}_i^n=(u_i^n,u_{i+1}^n,u_{i+2}^n)^T$, then we have

$$A_3\widetilde{U}_i^{n+1} = B_3\widetilde{U}_i^{n} + \widetilde{F}_i^{n}, \tag{14}$$

here $\widetilde{F}_i^n = \left(6ru_{i-1}^{n+1}, 0, 6ru_{i+1}^{n}\right)^T$,

$$A_3 = \begin{pmatrix} 5+9r & 1-3r & 0 \\ 1-6r & 10+12r & 1-6r \\ 0 & 1-3r & 5+3r \end{pmatrix}, \ B_3 = \begin{pmatrix} 5-3r & 1+3r & 0 \\ 1+6r & 10-12r & 1+6r \\ 0 & 1+3r & 5-9r \end{pmatrix}.$$

Thus we have

$$\widetilde{U}_i^{n+1} = A_3^{-1}(B_3\widetilde{U}_i^{n} + \widetilde{F}_i^{n}).$$

Applying the basic point groups above, we construct the alternating group method in the case of two conditions as follows:

In the first condition, we let $m-1=6s$, here s is an integer. First at the $(n+1)$-th time level, we divide all of the $m-1$ inner grid points into s "$\kappa 1$" groups, and (12) are used in each group. Second at the $(n+2)$-th time level, we will have $(s+1)$ point groups. "$\kappa 3$" group are applied to get the solution of the left three grid points $(1, n+2)$, $(2, n+2)$, $(3, n+2)$. (12) are used in the following s "$\kappa 1$" groups, while "$\kappa 2$" are used in the right three grid points $(m-3, n+2)$, $(m-2, n+2)$, $(m-1, n+2)$.

Let $U^n = (u_1^n,\ u_2^n,\ \ldots,\ u_{m-1}^n)^T$, then we can denote the alternating group explicit method I as follows:

$$\begin{cases} AU^{n+1} = BU^n + C_1^n, \\ \widehat{A}U^{n+2} = \widehat{B}U^{n+1} + C_2^n, \end{cases} \tag{15}$$

here C_1^n and C_2^n are known vectors relevant to the boundary, while $A,\ B,\ \widehat{A},\ \widehat{B}$ are all $(m-1)\times(m-1)$ matrices.

$$\begin{aligned} &C_1^n = (6ru_0^n, 0, \ldots, 0, 6ru_m^n)^T, \quad C_2^n = (6ru_0^{n+2}, 0, \ldots, 0, 6ru_m^{n+2})^T, \\ &A = \mathrm{diag}(A_1,\ A_1,\ \ldots,\ A_1,\ A_1), \quad B = \mathrm{diag}(B_2,\ \overline{B}_1,\ \ldots,\ \overline{B}_1,\ B_3), \\ &\widehat{A} = \mathrm{diag}(A_3,\ A_1,\ \ldots,\ A_1,\ A_2), \quad \widehat{B} = \mathrm{diag}(\overline{B}_1,\ \overline{B}_1,\ \ldots,\ \overline{B}_1,\ \overline{B}_1). \end{aligned}$$

Here

$$\overline{B}_1 = \begin{pmatrix} 5-3r & 1+3r & 0 & 0 & 0 & 0 \\ 1+6r & 10-12r & 1+6r & 0 & 0 & 0 \\ 0 & 1+3r & 5-9r & 6r & 0 & 0 \\ 0 & 6r & 5-9r & 1+3r & 0 & 0 \\ 0 & 0 & 0 & 1+6r & 10-12r & 1+6r \\ 0 & 0 & 0 & 0 & 1+3r & 5-3r \end{pmatrix}.$$

In the following we will try to construct the alternating method under the condition of $m-1=6s+3$: First at the $(n+1)$-th time level, we will have $s+1$ point groups. "$\kappa 2$" are used at the right three inner grid points, while the left $6s$ inner grid points are divided into s groups, and "$\kappa 1$" are used in each group. Second at the $(n+2)$-th time level, we are still to have $s+1$ point groups. "$\kappa 3$"

are used at the left three inner grid points, while the right $6s$ inner grid points are divided into s groups, and "$\kappa 1$" are used in each group. Thus the alternating group method II is established by alternating use of the schemes (8)-(11) and (3) in the two time levels:

$$\begin{cases} \widetilde{A}U^{n+1} = \widetilde{B}U^n + \widetilde{C}_1^n, \\ \widetilde{\widetilde{A}}U^{n+2} = \widetilde{\widetilde{B}}U^{n+1} + \widetilde{C}_2^n, \end{cases} \tag{16}$$

here $\widetilde{C}_1^n$ and $\widetilde{C}_2^n$ are known vectors relevant to the boundary, while $\widetilde{A}$, $\widetilde{B}$, $\widetilde{\widetilde{A}}$, $\widetilde{\widetilde{B}}$ are all $(m-1)\times(m-1)$ matrices.

$$\begin{aligned} &\widetilde{C}_1^n = (6ru_0^n, 0, \ldots, 0, 6ru_m^{n+1})^T, \quad \widetilde{C}_2^n = (6ru_0^{n+2}, 0, \ldots, 0, 6ru_m^{n+1})^T, \\ &\widetilde{A} = \text{diag}(A_1,\ A_1,\ \ldots,\ A_1,\ A_2), \quad \widetilde{B} = \text{diag}(B_2,\ \overline{B}_1, \ldots, \overline{B}_1,\ B_1), \\ &\widetilde{\widetilde{A}} = \text{diag}(A_3,\ A_1,\ \ldots,\ A_1,\ A_1), \quad \widetilde{\widetilde{B}} = \text{diag}(\overline{B}_1,\ \overline{B}_1,\ \ldots,\ \overline{B}_1,\ B_3). \end{aligned}$$

3. Stability and Error Analysis

In order to verify the stability of (15) and (16), we present the following lemma [4]:

Lemma 3.1. *If $M = (m_{ij})$ is a $n \times n$ diagonal dominant L-matrix, while $N = (n_{ij})$ is a $n \times n$ nonnegative definite matrix, then it holds:*

$$\min_i \left(\sum_j n_{ij} / \sum_j m_{ij} \right) \le \rho(M^{-1}N) \le \|M^{-1}N\|_\infty \le \max_i \left(\sum_j n_{ij} / \sum_j m_{ij} \right). \tag{17}$$

Theorem 3.2. *The alternating group method denoted by* (15) *is unconditionally stable.*

Proof. From (15) we have $U^{n+2} = GU^n + C^n$, here $G = \widehat{A}^{-1}\widehat{B}A^{-1}B$ is the growth matrix. $C^n = \widehat{A}^{-1}\widehat{B}A^{-1}C_1^n + \widehat{A}^{-1}C_2^n$. From the construction of the matrices above we can see that A and $\widehat{A}$ are both strictly diagonally dominant L-matrices, while B and $\widehat{B}$ are both nonnegative definite real matrices. Furthermore we have

$$\rho(\widehat{A}^{-1}\widehat{B}) \le 1,\ \rho(A^{-1}B) \le 1.$$

Then we have $\rho(G) = \rho(\widehat{A}^{-1}\widehat{B}A^{-1}B) \le \rho(\widehat{A}^{-1}\widehat{B})\rho(A^{-1}B) \le 1$, which shows that the alternating group method (15) is of unconditional stability. ∎

Analogously we have the following theorem.

Theorem 3.3. *The alternating group method* (16) *is also unconditionally stable.*

Considering $\left(\frac{\partial^k u}{\partial t^k}\right)_i^n = \left(\frac{\partial^{2k} u}{\partial x^{2k}}\right)_i^n$, applying Taylor formula to (8)-(11) we can easily obtain that the truncation error is $O(\tau^2+\tau h+\tau^2 h^2+h^4)$ respectively, and alternating use of (8)-(11) can lead to counteraction of the truncation error for the items containing τh and $\tau^2 h^2$. Then we can denote the truncation error of (15) as $O(\tau^2+h^4)$, which shows (15) is compatible with (1).

According to Lax theorem, (15) is convergent under the fact of unconditional stability.

Analogously (16) is also convergent. So we have:

Theorem 3.4. *The alternating group methods denoted by* (15) *and* (16) *are convergent, and have the truncation error order* $O(\tau^2+h^4)$.

4. Numerical Experiments

Example 4.1. We consider the following homogeneous boundary value problem:

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ 0 \le x \le 1, \ 0 \le t \le T, \\ u(x,0) = \sin(\pi x), \\ u(0,t) = 0, \ u(1,t) = 0. \end{cases} \tag{18}$$

The exact solution of (18) is denoted in [2] as below:

$$u(x,t) = e^{-\pi^2 t}\sin(\pi x).$$

Let A.E.=$|u_i^n - u(x_i,t_n)|$ and P.E.=$\frac{|u_i^n - u(x_i,t_n)|}{u(x_i,t_n)}$ denote maximum absolute error and relevant error respectively. We compare the numerical results of the presented alternating group method in this paper with the results from center-difference implicit scheme (3) and the method from [2] in Table 1 and Table 2. Let t_1/t_2 denotes the ratio of running time between the AGE method in this paper and center-difference implicit scheme (3). We performed the numerical experiment in the same conditions.

Table 1. Results of Comparison at $m = 25$

	$\tau = 10^{-3}$, $t = 100\tau$	$\tau = 10^{-3}$, $t = 500\tau$	$\tau = 10^{-4}$, $t = 100\tau$	$\tau = 10^{-4}$, $t = 500\tau$
A.E.	5.258×10^{-4}	4.758×10^{-5}	4.770×10^{-5}	9.610×10^{-6}
A.E.[2]	1.653×10^{-3}	1.548×10^{-4}	1.117×10^{-4}	5.193×10^{-5}
P.E.	1.587×10^{-2}	6.852×10^{-2}	6.833×10^{-3}	3.267×10^{-3}
P.E.[2]	4.664×10^{-1}	2.179	1.785×10^{-2}	6.787×10^{-2}
t_1/t_2	0.251	0.255	0.245	0.262

Table 2. Results of Comparison at $m = 34$

	$\tau = 10^{-3}$, $t = 100\tau$	$\tau = 10^{-3}$, $t = 500\tau$	$\tau = 10^{-4}$, $t = 100\tau$	$\tau = 10^{-4}$, $t = 500\tau$
A.E.	9.576×10^{-4}	2.881×10^{-5}	6.112×10^{-6}	2.472×10^{-5}
$A.E.^{[2]}$	3.513×10^{-3}	3.302×10^{-4}	5.802×10^{-5}	1.958×10^{-4}
P.E.	3.284×10^{-2}	1.313×10^{-1}	3.565×10^{-3}	6.549×10^{-3}
$P.E.^{[2]}$	9.737×10^{-1}	4.626	1.626×10^{-2}	4.249×10^{-2}
t_1/t_2	0.158	0.164	0.154	0.173

Example 4.2. We consider the following nonhomogeneous boundary value problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = 0.01\dfrac{\partial^2 u}{\partial x^2},\ 0 \leq x \leq 1,\ 0 \leq t \leq T, \\ u(x,0) = \cos(\pi x), \\ u(0,t) = e^{-0.01\pi^2 t},\ u(1,t) = -e^{-0.01\pi^2 t}. \end{cases} \tag{19}$$

The exact solution of (19) is denoted by

$$u(x,t) = e^{-0.01\pi^2 t}\cos(\pi x).$$

Let $\|E_1\|_\infty = \max|u_i^n - u(x_i,t_n)|$, $\|E_2\|_\infty = \max|(u_i^n - u(x_i,t_n))/u(x_i,t_n)|$, $i = 1,2,\ldots,m-1$. The numerical results is listed in Table 3.

Table 3. Results of Comparison at $m = 49$

	$\tau = 10^{-1}$, $t = 10\tau$	$\tau = 10^{-1}$, $t = 20\tau$	$\tau = 10^{-1}$, $t = 50\tau$	$\tau = 10^{-1}$, $t = 100\tau$
$\|\|E_1\|\|_\infty$	4.698×10^{-4}	6.315×10^{-4}	7.028×10^{-4}	1.015×10^{-3}
$\|\|E_1\|\|_\infty[2]$	6.837×10^{-3}	2.361×10^{-2}	2.693×10^{-2}	8.013×10^{-2}
$\|\|E_2\|\|_\infty$	3.395×10^{-2}	3.167×10^{-2}	2.674×10^{-2}	4.247×10^{-1}
$\|\|E_2\|\|_\infty[2]$	7.412×10^{-1}	1.846	2.131	3.256
t_1/t_2	0.114	0.119	0.131	0.126

From the results of Tables 1, 2, 3 we can see that the numerical solution for the presented method is of higher accuracy than the original AGE method in [2], which is obvious even in the case of large τ. Furthermore, for its intrinsic parallelism, the AGE method in this paper can shorten the running computing time in comparison with the fully implicit scheme, and the effect becomes obvious when the amount of grid points increases.

5. Conclusions

In this paper, we present a class of high order alternating group explicit method, which is verified to be unconditionally stable and convergent. Considering the absolute stability of this method, it doesn't lead to numerical vibration using any time step size and spatial step size in computation. Based on the characters, it is an effective parallel method in solving large equation sets.

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