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Rings over which Polynomial Rings are Semi-commutative*

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Abstract. We introduce *SSC* rings and show that these rings and reversible rings are independent subclasses of the class of semi-commutative rings. We develop their basic properties: this class is closed under subdirect products, polynomial extensions, Laurent polynomial extensions. We also provide construction techniques and examples of *SSC* rings. At last, we obtain that R is right (resp. left) strongly Hopfian if and only if the polynomial factor ring $R[x]/(x^{n+1})$ is right (resp. left) strongly Hopfian, where (x^{n+1}) is the ideal generated by x^{n+1} and n is any nonnegative number, in case the ring R satisfies the property (P).

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1. Introduction

Throughout this paper, all rings are associative with identity $1(\neq 0)$. For a ring R, the notations $\gamma_R(-)$ and $\iota_R(-)$ are used for the right and left, respectively, annihilators over R.

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A ring R is called semi-commutative, if for all $a, b \in R$, $ab = 0$ implies $aRb = 0$. This is equivalent to the usual definition by [14, Lemma 1.2] or [5, Lemma 1]. Properties, examples and counterexamples of semi-commutative rings were given in Huh, Lee and Smoktunowicz [5], Kim and Lee [7], Liu [9] and Yang [15]. A generalization of semi-commutative rings was investigated in Liu and Zhao [10]. In [2], Cohn introduced the notion of a reversible ring. A ring R is said to be reversible, if whenever $a, b \in R$ satisfy $ab = 0$, then $ba = 0$. Reduced rings (i.e., rings with no nonzero nilpotent elements) and commutative rings are clearly reversible, and reversible rings are semicommutative by [8, Proposition 1.3]. i.e., the following implications hold:

$$\text{reduced (also, commutative)} \Rightarrow \text{reversible} \Rightarrow \text{semi-commutative}$$

In general, each of these implications are irreversible (see [11]). In [3, Corollary 2.3] it was claimed that all semi-commutative rings are McCoy. However, Hirano's claim assumed that if R is semi-commutative then $R[x]$ is semi-commutative, but this was later shown to be false in [5, Example 2]. Moreover, Nielsen [12] gave an example to show that a semi-commutative ring R need not be right McCoy, we also prove that the polynomial ring $R[x]$ over it actually is not semi-commutative. Thus it is a natural and an interesting work to consider these semi-commutative rings over which polynomial rings are semi-commutative and we call them strongly semi-commutative (SSC for short), i.e., a ring R is called SSC, if whenever polynomials $f(x)$, $g(x)$ in $R[x]$ satisfy $f(x)g(x) = 0$, then $f(x)R[x]g(x) = 0$. Clearly the following implications hold:

$$\text{reversible (also, } SSC) \Rightarrow \text{semi-commutative}$$

We give examples to show that reversible rings and SSC rings are independent subclasses of the class of semi-commutative rings. It is also proved that a ring R is SSC if and only if its polynomial ring $R[x]$ is SSC, if and only if its Laurent polynomial ring $R[x, x^{-1}]$ is SSC. At last, we prove that if R is a ring satisfying the property (P), then R is a right (resp. left) strongly Hopfian R-module if and only if $R[x]/(x^{n+1})$ is a right (resp. left) strongly Hopfian $R[x]/(x^{n+1})$-module, where (x^{n+1}) is the ideal generated by x^{n+1} and n is any nonnegative number.

2. Definitions and Examples

Recall that a ring R is said to be *right McCoy* if the equation $f(x)g(x) = 0$ over $R[x]$, where $f(x), g(x) \neq 0$, implies there exists a nonzero $r \in R$ with $f(x)r = 0$. We define *left McCoy* rings similarly. If a ring is both left and right McCoy we say that the ring is a McCoy ring [12]. In [3, Corollary 2.3] it was claimed that all semi-commutative rings were McCoy. However, Hirano's claim assumed that if R is semi-commutative then $R[x]$ is semi-commutative, but this was later shown to be false in [5, Example 2]. Thus it is natural to introduce the following notion.

Definition 2.1. A ring R is called strongly semi-commutative (SSC for short), if whenever polynomials $f(x)$, $g(x)$ in $R[x]$ satisfy $f(x)g(x) = 0$, then

$$f(x)R[x]g(x) = 0.$$

Obviously, R is SSC if and only if $R[x]$ is semi-commutative.

Clearly, the class of SSC rings is closed under subrings and direct products. Also, any reduced or commutative rings are SSC, any SSC ring is obviously semi-commutative, thus by Hirano's claim in [3, Corollary 2.3], it is easy to get that all SSC rings are McCoy. However Nielsen [12] gave an example to show that a semi-commutative ring R need not be right McCoy, we also prove that the polynomial ring $R[x]$ over it actually is not semi-commutative as follows.

Example 2.2. Let $A = Z_2 < a_0, a_1, a_2, a_3, b_0, b_1 >$ be the free associative algebra (with 1) over Z_2 generated by six indeterminates (as labeled above). Let I be the ideal generated by the following relations:

$$< a_0b_0, a_0b_1 + a_1b_0, a_1b_1 + a_2b_0, a_2b_1 + a_3b_0, a_3b_1,$$
$$a_0a_j(0 \leq j \leq 3), a_3a_j(0 \leq j \leq 3), a_1a_j + a_2a_j(0 \leq j \leq 3),$$
$$b_ib_j(0 \leq i, j \leq 1), b_ia_j(0 \leq i \leq 1, 0 \leq j \leq 3) > .$$

Let $R = A/I$. Nielsen [12] proved that R is semi-commutative and not right McCoy but left McCoy. Further, we will demonstrate that R is not SSC.

Proof. Put $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $g(x) = b_0 + b_1x$. The first row of relations in I guarantees that $f(x)g(x) = 0$ in $R[x]$, however we have that $f(x)a_0g(x) \neq 0$ since $a_1a_0b_1 + a_0a_0b_2 \notin I$. ∎

Since reversible rings and SSC rings are semi-commutative, one may suspect that any reversible ring is SSC or any SSC ring is reversible. But the conjectures are not true in general as follows.

Example 2.3. Let S be a reduced ring and

$$R = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \middle| \; a, b, c, d \in S \right\}.$$

Then R is semi-commutative by [7, Proposition 1.2] and Armendariz by [6, Proposition 2], hence R is SSC, by Proposition 3.4. But it is not reversible by [6, Example 1.5].

Example 2.4. We refer to the argument [7, Example 2.1]. Let Z_2 be the field of integers modulo 2 and $A = Z_2[a_0, a_1, a_2, b_0, b_1, b_2, c]$ be the free algebra of polynomials with zero constant terms in non-commuting indeterminates

$a_0, a_1, a_2, b_0, b_1, b_2, c$ over Z_2. Note that A is a ring without identity and consider an ideal of the ring $Z_2 + A$, say I, generated by

$$a_0b_0, a_0b_1 + a_1b_0, a_0b_2 + a_1b_1 + a_2b_0, a_1b_2 + a_2b_1, a_2b_2, a_0rb_0, a_2rb_2,$$

$$b_0a_0, b_0a_1 + b_1a_0, b_0a_2 + b_1a_1 + b_2a_0, b_1a_2 + b_2a_1, b_2a_2, b_0ra_0, b_2ra_2,$$

$$(a_0 + a_1 + a_2)r(b_0 + b_1 + b_2), (b_0 + b_1 + b_2)r(a_0 + a_1 + a_2), \text{ and } r_1r_2r_3r_4.$$

Where $r, r_1, r_2, r_3, r_4 \in A$. Then clearly $A^4 \in I$. Next let $R = (Z_2 + A)/I$ and consider $R[x] \cong ((Z_2 + A)[x])/(I[x])$. Notice that

$$(a_0 + a_1x + a_2x^2)(b_0 + b_1x + b_2x^2) \in I[x],$$

but

$$(a_0 + a_1x + a_2x^2)c(b_0 + b_1x + b_2x^2) \notin I[x]$$

because $a_0cb_1 + a_1cb_0 \notin I$, hence R is not SSC since $R[x]$ is not semicommutative. But R is reversible by [7, Example 2.1].

3. SSC Rings and Extensions

In this section we observe the basic properties of SSC rings and extensions of rings.

Proposition 3.1. *Let R be a ring, e a central idempotent of R, Δ be a multiplicative closed subset consisting of central regular elements of R. Then the following statements are equivalent:*

(i) *R is SSC.*

(ii) *eR and $(1 - e)R$ are SSC.*

(iii) *$\Delta^{-1}R$ is SSC.*

Proof. (i)⇔(ii). It is straightforward since subrings and direct products of SSC rings are SSC.

(iii)⇒(i). It is obvious.

(i)⇒(iii). Let $f(x) = \sum_{i=0}^{m} u_i^{-1}a_ix^i$, $g(x) = \sum_{j=0}^{n} v_j^{-1}b_jx^j \in \Delta^{-1}R[x]$ satisfy $f(x)g(x) = 0$. For any $h(x) = \sum_{k=0}^{l} w_k^{-1}c_kx^k \in \Delta^{-1}R[x]$, we have that $\tilde{f}(x) = (u_mu_{m-1}\cdots u_0)f(x)$, $\tilde{g}(x) = (v_nv_{n-1}\cdots v_0)g(x)$ and $\tilde{h}(x) = (w_lw_{l-1}\cdots w_0)h(x) \in R[x]$ and $\tilde{f}(x)\tilde{g}(x) = 0$, so $\tilde{f}(x)\tilde{h}(x)\tilde{g}(x) = 0$ since R is SSC. Thus we have $f(x)h(x)g(x) = 0$ since all $u_i, v_j, w_k, i = 0, 1, \ldots, m, j = 0, 1, \ldots, n, k = 0, 1, \ldots, l$ are regular and central. ■

Let $\{S_\lambda\}_{\lambda\in\Lambda}$ be a family of rings indexed by a set Λ, $\prod_\Lambda S_\lambda = S$ be the Cartesian product of $\{S_\lambda\}_{\lambda\in\Lambda}$. A ring R is called the subdirect product of the rings $\{S_\lambda\}_{\lambda\in\Lambda}$, if there exists an injective ring homomorphism $\phi : R \to S = \prod_\Lambda S_\lambda$ such that $\pi_\lambda\phi$ is an surjective ring homomorphism for any $\lambda \in \Lambda$, where each $\pi_\lambda : S = \prod_\Lambda S_\lambda \to S_\lambda$ is the projection onto the λ-th component. It is easy to show that R is the subdirect product of a family rings if and only if there exists a family of ideals $\{I_\lambda\}_{\lambda\in\Lambda}$ of R such that R is the subdirect product of $\{R/I_\lambda\}_{\lambda\in\Lambda}$, where $\{I_\lambda\}_{\lambda\in\Lambda}$ satisfy $\bigcap_\Lambda I_\lambda = 0$. Direct products and direct sums of rings are all examples of subdirect products of rings.

Proposition 3.2. *Let R be a subdirect product of SSC rings. Then R is SSC.*

Proof. Let $I_\lambda(\lambda \in \Lambda)$ be ideals of R such that every R/I_λ is SSC and $\bigcap_{\lambda\in\Lambda} I_\lambda = 0$. Suppose that $f(x) = \sum_{i=0}^m a_i x^i$, $g(x) = \sum_{j=0}^n b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$. For any $h(x) = \sum_{k=0}^l c_k x^k \in R[x]$, we have that $\overline{f}(x)\overline{h}(x)\overline{g}(x) = 0$ in $(R/I_\lambda)[x]$ for each $\lambda \in \Lambda$ since R/I_λ is SSC. So $\sum_{i+j+k=t} a_i c_k b_j \in I_\lambda$ for $t = 0, 1, \ldots, m+n+l$ and any $\lambda \in \Lambda$, which implies that $\sum_{i+j+k=t} a_i c_k b_j = 0$ for $t = 0, 1, \ldots, m+n+l$ since $\bigcap_{\lambda\in\Lambda} I_\lambda = 0$. Thus we obtain $f(x)h(x)g(x) = 0$. ■

Corollary 3.3. *Let R be a semi-commutative ring and suppose that $Z(R)$ contains an infinite subring whose nonzero elements are regular in R. Then R is SSC.*

Proof. It is well-known that $R[x]$ is a subdirect product of R's, under given conditions. Using Proposition 3.2, it is easy to check that $R[x]$ is semi-commutative and we get that R is SSC. ■

A ring R is called Armendariz if whenever polynomials $f(x) = \sum_{i=0}^m a_i x^m$, $g(x) = \sum_{j=0}^n b_j x_n \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_i b_j = 0$ for each i, j (see [5, 6] for detail). Rege-Chhawchhaaria showed that commutative (hence semi-commutative) rings need not to be Armendariz in [13, Example 3.2]. Conversely Huh, Lee and Smoktunowicz [5] gave a ring which is Armendariz but not semi-commutative. However we have the following result.

Proposition 3.4. *Let R be an Armendariz ring. If R is a semi-commutative ring, then R is SSC.*

Proof. Suppose that $f(x) = \sum_{i=0}^m a_i x^i$, $g(x) = \sum_{j=0}^n b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$. Then since R is Armendariz, each $a_i b_j$ is zero, additionally R is semi-commutative, therefore $a_i r b_j = 0$ for any element r in R for all i, j. Now it is easy to check that $f(x)h(x)g(x) = 0$ for any $h(x) = \sum_{k=0}^l c_k x^k \in R[x]$. ■

Since reversible rings are semi-commutative, the following corollary is clear.

Corollary 3.5. *Let R be an Armendariz ring. If R is a reversible ring, then R is a SSC ring.*

A ring is called *locally finite* if every finite subset in it generates a finite semigroup multiplicatively. Finite rings are clearly locally finite, and an algebra closure of a finite field is locally finite but not finite.

Corollary 3.6. *Locally finite Armendariz rings are SSC and especially finite Armendariz rings are SSC.*

Proof. Locally finite Armendariz rings are semi-commutative by [5, Proposition 16], so it is SSC by Proposition 3.4. ■

Anderson and Camillo [1, Theorem 2] showed that a ring R is Armendariz if and only if $R[x]$ is Armendariz. It is obvious that R is reduced if and only if $R[x]$ is reduced. We have known that $R[x]$ may not be semi-commutative when R is semi-commutative. It is natural to suspect that if R is SSC, then $R[x]$ is SSC. So we have the following results.

Theorem 3.7. *Let R be a ring. Then the following statements are equivalent:*

(i) *R is SSC.*

(ii) *$R[x]$ is SSC.*

(iii) *$R[x,x^{-1}]$ is SSC.*

Proof. (i)⇒(ii) The idea of the proof comes from [1, Theorem 2]. Let $f(y), g(y) \in (R[x])[y]$ such that $f(y)g(y) = 0$. Let

$$f(y) = f_0 + f_1 y + \cdots + f_p y^p, \quad \text{where} \quad f_i = \sum_{s=0}^{m_i} a_s^{(i)} x^s,$$

$$g(y) = g_0 + g_1 y + \cdots + g_q y^q, \quad \text{where} \quad g_j = \sum_{t=0}^{m_j} b_t^{(j)} x^t,$$

$$h(y) = h_0 + h_1 y + \cdots + h_n y^n \in (R[x])[y], \quad \text{where} \quad h_k = \sum_{r=0}^{m_k} a_r^{(k)} x^r,$$

and

$$l = \deg(f_0) + \cdots + \deg(f_p) + \deg(g_0) + \cdots + \deg(g_q) + \deg(h_0) + \cdots + \deg(h_n),$$

where degree is as polynomials in x and the degree of the zero polynomial is taken to be 0. Then, by evaluating at x^l, we obtain polynomials $\tilde{f}(x) = f(x^l), \tilde{g}(x) = g(x^l)$ and $\tilde{h}(x) = h(x^l)$ whose coefficients are all f_i's, g_j's and h_k's, respectively. Also, since $f(y)g(y) = 0$ and x commutes with elements of R, we have that $\tilde{f}(x)\tilde{g}(x) = 0$, thus $f(y)h(y)g(y) = \tilde{f}(x)\tilde{h}(x)\tilde{g}(x) = 0$ since R is SSC, which implies $R[x]$ is SSC.

(ii)⇒(iii). It follows from Proposition 3.1.

(iii)⇒(i). Any subring of a SSC ring is again SSC. ■

Corollary 3.8. *Let R be a SSC ring and $\{x_\alpha\}$ any set of commuting indeterminates over R. Then any subring of $R[\{x_\alpha\}]$ is SSC.*

Proof. Let $f(y), g(y) \in R[\{x_\alpha\}][y]$ with $f(y)g(y) = 0$. For any $h(y) \in R[\{x_\alpha\}][y]$, we have that

$$f(y), g(y), g(y) \in R[\{x_{\alpha_1}, x_{\alpha_2}, \cdots, x_{\alpha_n}\}][y]$$

for some finite subset $\{x_{\alpha_1}, x_{\alpha_2}, \cdots, x_{\alpha_n}\} \subseteq \{x_\alpha\}$. By induction, the ring $R[\{x_{\alpha_1}, x_{\alpha_2}, \cdots, x_{\alpha_n}\}][y]$ is SSC, so we have that $f(y)h(y)g(y) = 0$. Hence $R[\{x_\alpha\}]$ is SSC and thus so is any subring of $R[\{x_\alpha\}]$. ∎

Examples of SSC rings are given in the following which also show that SSC rings are not reduced in general.

Example 3.9. Let R be a ring and n any positive integer. If R is reduced, then $R[x]/(x^n)$ is SSC, where (x^n) is the ideal generated by x^{n+1} and n is a positive integer.

Proof. By Corollary 3.5, $R[x]/(x^n)$ is SSC since it is reversible [7, Proposition 2.5] and Armendariz [1, Theorem 5]. ∎

Given a ring R and a bimodule ${}_RM_R$, the trivial extension of R by M is the ring $T(R, M) = R \bigoplus M$ with the usual addition and the following multiplication:

$$(r_1, m_1)(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2).$$

This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$, $m \in M$ and the usual matrix operations are used.

Example 3.10. Let R be a ring and $T = R \bigoplus R$ be the trivial extension of R by R. If R is reduced, then T is SSC.

Proof. It is clear since $T = R \bigoplus R \cong R[x]/(x^2)$ is SSC. ∎

Considering Example 3.10, we may suspect that if a ring R is SSC, then $T(R, R)$ is SSC. However, this is not true from [7, Example 1.7] and easy check. One may still conjecture that a ring R is SSC if for any nonzero proper ideal I of R, R/I and I are SSC, where I is considered as a ring without the identity. Also the following example erases the possibility.

Example 3.11. Let S be a division ring and

$$R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \middle| \; a, b, c \in S \right\}.$$

Then R is not SSC since it is not semi-commutative [5, Example 5]. Notice that R has only the following nonzero proper ideals:

$$K_1 = \begin{pmatrix} S & S \\ 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & S \\ 0 & S \end{pmatrix}, \quad K_3 = \begin{pmatrix} 0 & S \\ 0 & 0 \end{pmatrix}.$$

The same methods as in [5, Example 5] can be used to prove that K_i's and R/K_i's are all SSC.

Huh and Lee showed that if R/I is semi-commutative for some reduced ideal I, then R is semi-commutative [5, Theorem 6]. In fact, this result can be strengthened as the following:

Proposition 3.12. *Suppose that R/I is SSC for some ideal I of a ring R. If I is reduced, then R is SSC.*

Proof. Notice that $I[x]$ is reduced and $R[x]/I[x] \simeq (R/I)[x]$ is semi-commutative, therefore $R[x]$ is semi-commutative by [5, Theorem 6]. So R is SSC. ■

Recall that a ring R is called right *strongly Hopfian*, if for any element $r \in R$, the chain $\ell_R(r) \subseteq \ell_R(r^2) \subseteq \cdots$ stabilizes [4]. Clearly, any right noetherian ring is right strongly Hopfian, and the concept of left strongly Hopfian is defined similarly. Hmaimou, Kaidi, and Sánchez Campos [4] proved that $R[x]$ is strongly Hopfian if and only if R is strongly Hopfian in case R is a commutative ring.

Example 3.9 shows that the polynomial factor ring over a ring R, $R[x]/(x^n)$, may be SSC (hence semi-commutative), where (x^n) is the ideal generated by x^n and n is a positive integer. It is clear that the polynomial factor rings over commutative rings are also commutative, so examples of such rings are extensive. Next we will consider strongly Hopficiy of rings over which the factor rings are also semi-commutative. Firstly, we give the following definition.

Definition 3.13. We call a ring R satisfying the property (P), if $R[x]/(x^{n+1})$ is semi-commutative for any nonnegative number integer n.

Clearly, rings that satisfy the property (P) are semi-commutative.

Theorem 3.14. *Let R be a ring satisfying property (P), then the following conditions are equivalent:*

(i) *R is a right (resp. left) strongly Hopfian R-module.*

(ii) *$R[x]/(x^{n+1})$ is a right (resp. left) strongly Hopfian $R[x]/(x^{n+1})$-module for any nonnegative number n.*

Proof. (i)⇒(ii). Put $u = x + (x^{n+1})$, then we have $u^{n+i} = 0$ for every $i \geq 1$. We denote $R[x]/(x^{n+1})$ by $R[u]$. For any $p(u) = r_0 + r_1 u + \cdots + r_n u^n \in R[u]$, since R is right strongly Hopfian, there exists a positive integer l such that $\ell_R(r_0^{l+1}) = \ell_R(r_0^l)$. Let $k = \max\{l, n+1\}$. In the following we show that $\ell_{R[u]}(p(u)^{2k+1}) = \ell_{R[u]}(p(u)^{2k})$.

Let $f(u) = a_0 + a_1 u + \cdots + a_n u^n \in \ell_{R[u]}(p(u)^{2k+1})$. It is clear that $a_0 \in \ell_R(r_0^k)$. Since $f(u)p(u)^{2k+1} = 0$ and $R[x]/(x^{n+1})$ is semi-commutative, we have $f(u)r_0^k p(u)^{2k+1} = 0$ and get $a_1 \in \ell_R(r_0^k)$. In this way, we have every $a_i \in \ell_R(r_0^k)$ for $i = 0, 1, \ldots, n$. We denote $p(u)$ by the sum of two elements r_0 and $q(u) =$

$r_1u + \ldots + r_nu^n$. Using semi-commutativity of $R[x]/(x^{n+1})$, it can be easily checked that $f(u)p(u)^{2k} = f(u)[r_0 + q(u)]^{2k} = 0$ because all the terms in the development of $[r_0 + q(u)]^{2k}$ with powers of r_0 less than k have powers of $q(u)$ more than n. Thus we obtain that $\ell_{R[x]}(p(u)^{2k+1}) = \ell_{R[x]}(p(u)^{2k})$.

(ii)⇒(i). It is obvious and we omit it. ■

Since any commutative (reduced) ring satisfies the property (P), the following result is an easy consequence of Theorem 3.14.

Corollary 3.15. *Let R be a commutative* (*reduced*) *ring, then the following conditions are equivalent:*

(i) *R is a right* (*resp. left*) *strongly Hopfian R-module.*

(ii) *$R[x]/(x^{n+1})$ is a right* (*resp. left*) *strongly Hopfian $R[x]/(x^{n+1})$-module for any nonnegative number n.*

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