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A Novel Exponential Stability Condition for a Class of Hybrid Neural Networks with Time-varying Delay

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Abstract. This paper proposes a switching design for exponential stability of a class of hybrid neural networks with time-varying delay and various activation functions. By using time-varying delay Lyapunov-Krasovskii functional, a switching rule for the exponential stability is designed in terms of the solution of Riccati-type equations. The approach allows for computation of the bounds that characterize the exponential stability rate of the solution. An example is given to illustrate the result.

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Key words: Neural networks, switched systems, exponential stability, time delay, Lyapunov function, Riccati equation.

1. Introduction

In recent years, neural networks have received increasing interest due to their wide range of applications in, for example, pattern recognition, associative memory, and combinational optimization, see, e.g. [2, 4, 6] and the references therein. A switched system can be characterized by the following differential equation:

$$\dot{x}(t) = f_\sigma(x(t)), \quad t \geq 0,$$

where $\{f_\sigma : \sigma \in \mathcal{N}\}$ is a family of sufficiently regular functions from $\mathbb{R}^n$ to $\mathbb{R}^n$, which is parameterized by some index set $\mathcal{N}$, and $\sigma : [0, +\infty) \to \mathcal{N}$ is a piecewise constant function of time, named a switching signal. In specific situations, the value of $\sigma(.)$ at a given time t might depend on $x(t)$, or may be generated by using some more sophisticated techniques such as hybrid feedback with memory in the loop. The hybrid systems are composed of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching between the subsystems [9, 13, 15, 16].

For switched neural networks, based on the Lyapunov Krasovskii method and linear matrix inequality approach, some conditions for asymptotic stability of switched neural networks with time-invariant delays are proposed in [2, 5, 12, 18], with time-varying delays in [3, 7, 10, 11, 14]. However, the proposed stability criteria in the mentioned papers are derived for switched systems under arbitrary switching laws, which are restrictive and less applicable for almost switched systems [9, 16]. To the best of our knowledge, there has not been available in the literature a paper that addresses the problem of switching design for the stability of neural networks with time-varying delays, as dealt with in this paper.

In this paper, we propose a switching design for exponential stability of a class of hybrid neural networks with time-varying delay and various activation functions. By using appropriate Lyapunov-Krasovskii functional combined with Riccati equation approach, a new sufficient condition for the exponential stability of the system is established. The delay-dependent condition is formulated in terms of the solution of algebraic Riccati-type equations, which allow to compute simultaneously the two bounds that characterize the exponential stability rate of the solution. Compared to other stability criteria, our result has its own advantages. First, it deals with the switching design for exponential stability of a class of hybrid neural networks. Second, our result studies the case of neural networks systems with both time-varying-delays and various activations functions.

The remaining part of the paper is organized as follows. In Sec. 2, the model formulation and some preliminaries are given. The main result and an example illustrated the effectiveness of the proposed result are presented in Sec. 3. Finally, concluding remarks are made in Sec. 4.

2. Preliminaries

The following notations will be used throughout this paper.

• $\mathbb{R}^+$ denotes the set of all real non-negative numbers;

• $\mathbb{R}^n$ denotes the n-dimensional space with the scalar product $\langle ., . \rangle$ and the vector norm $\|.\|$;

• $\mathbb{R}^{n \times r}$ denotes the space of all matrices of $(n \times r)$-dimension;

• A^T denotes the transpose of A; I denotes the identity matrix;

• $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$;

• A matrix A is semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$;

• $C([a, b], \mathbb{R}^n)$ denotes the space of all $\mathbb{R}^n$-valued continuous functions on $[a, b]$;

Let us denote $x_t := \{x(t + s), s \in [-h, 0]\}$ the segment of the trajectory $x(t)$ with the norm $\|x_t\| = \sup_{t\in[-h,0]} \|x(t + s)\|$.

Consider the following switched neural networks with time-varying delay

$$\begin{aligned}&\dot{u}(t) = -A_\sigma u(t) + B_\sigma \bar{f}_\sigma(u(t)) + C_\sigma \bar{g}_\sigma(u(t - \tau(t))) + \mathcal{I}_\sigma,\\ &u(t) = \phi(t), \quad t \in [-\tau, 0],\end{aligned} \tag{1}$$

where $\sigma(.) : \mathbb{R}^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching function, which is piecewise constant function depending on the state at each time and will be designed; $u(t) = [u_1(t), u_2(t), ..., u_n(t)]^T \in \mathbb{R}^n$ is the state vector of the neural networks; n is the number of neurals, and

$$\begin{aligned}&\bar{f}_i(u(t)) = [\bar{f}_{i1}(u_1(t)), \bar{f}_{i2}(u_2(t)), \ldots, \bar{f}_{in}(u_n(t))]^T,\\ &\bar{g}_i(u(t)) = [\bar{g}_{i1}(u_1(t)), \bar{g}_{i2}(u_2(t)), \ldots, \bar{g}_{in}(u_n(t))]^T, \quad i = 1, 2, \ldots, N,\end{aligned}$$

are the neural activation functions; the diagonal matrix $A_i = \mathrm{diag}(a_{i1}, a_{i2}, \ldots, a_{in})$, $i = 1, 2, \ldots, N$ represents the self-feedback term, and the matrices $[B_i, C_i], i = 1, 2, \ldots, N$ denote, respectively, the connection weights and the discretely delayed connection weights; $\mathcal{I}_i = [\mathcal{I}_{i1}, \mathcal{I}_{i2}, \ldots, \mathcal{I}_{in}]^T, i = 1, 2, \ldots, N$, is the constant external input vector; the time-varying delay function $\tau(t)$ satisfies

$$0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \delta < 1.$$

The initial function $\phi(t) \in C([-\tau, 0], \mathbb{R}^n)$, with the uniform norm

$$||\phi|| = \max_{t\in[-\tau,0]} \|\phi(t)\|.$$

In the field of neural networks, a typical assumption is that the only activation function to be considered is continuous, differentiable, monotonically increasing, and bounded. However, in this paper we consider various activation functions and do not require the monotonicity and boundedness. Instead, throughout this paper, we assume the following assumption.

(H_1) The activation functions $\bar{f}_{ij}(u), \bar{g}_{ij}(u), i = 1, 2, \ldots, N, j = 1, 2, \ldots, n$ are globally Lipschitzian with the Lipschitz constants $a_{ij} > 0, b_{ij} > 0, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n :$

$$\begin{aligned}|\bar{f}_{ij}(\xi_1) - \bar{f}_{ij}(\xi_2)| &\leq a_{ij}|\xi_1 - \xi_2|, \forall \xi_1, \xi_2 \in \mathbb{R}\\ |\bar{g}_{ij}(\xi_1) - \bar{g}_{ij}(\xi_2)| &\leq b_{ij}|\xi_1 - \xi_2|, \forall \xi_1, \xi_2 \in \mathbb{R}.\end{aligned} \tag{2}$$

As usual, vector $u^* = [u_1^*, u_2^*, \ldots, u_n^*]^T$ denotes an equilibrium point of system (1). As it is well known that under the above assumption, there is an equilibrium point u^* for (1), then by setting $x(t) = u(t) - u^*$, the system (1) can be transformed to the system

$$\begin{aligned} &\dot{x}(t) = -A_\sigma x(t) + B_\sigma f_\sigma(x(t)) + C_\sigma g_\sigma(x(t-\tau(t)), \\ &x(t) = \phi(t), \quad t \in [-h, 0], \end{aligned} \tag{3}$$

where $\sigma(.) : \mathbb{R}^n \to \{1, 2, \ldots, N\}$, and

$$\begin{aligned} f_i(x(t)) &= [f_{i1}(x_1(t)), f_{i2}(x_2(t)), \ldots, f_{in}(x_n(t))]^T, \\ g_i(x(t)) &= [g_{i1}(x_1(t)), g_{i2}(x_2(t)), \ldots, g_{in}(x_n(t))]^T \end{aligned}$$

and

$$\begin{aligned} &f_{ij}(x_j) = \bar{f}_{ij}(x_j + u_j^*) - \bar{f}_{ij}(u_j^*), \\ &g_{ij}(x_j) = \bar{g}_{ij}(x_j + u_j^*) - \bar{g}_{ij}(u_j^*), i = 1, 2, \ldots, N, j = 1, 2, \ldots, n. \end{aligned}$$

Then, from the condition (2), we have $f_{ij}(0) = 0, g_{ij}(0) = 0, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n$, and

$$\begin{aligned} |f_{ij}(\xi_1) - f_{ij}(\xi_2)| \le & a_{ij}|\xi_1 - \xi_2|, \forall \xi_1, \xi_2 \in \mathbb{R} \\ |g_{ij}(\xi_1) - g_{ij}(\xi_2)| \le & b_{ij}|\xi_1 - \xi_2|, \forall \xi_1, \xi_2 \in \mathbb{R}. \end{aligned} \tag{4}$$

Definition 2.1. Given $\beta > 0$. The system (3) is β-exponentially stable if there exists a switching function $\sigma(.)$ such that any solution $x(t, \phi)$ of the system satisfies

$$\exists N > 0 : \quad \|x(t, \phi)\| \le \gamma e^{-\beta t}\|\phi\|, \quad \forall t \in \mathbb{R}^+.$$

Definition 2.2. [17] The system of matrices $\{L_i\}, i = 1, 2, \ldots, N$, is said to be strictly complete if for every $x \in \mathbb{R}^n \backslash \{0\}$ there is $i \in \{1, 2, \ldots, N\}$ such that $x^T L_i x < 0$.

Let us define

$$\Omega_i = \{x \in \mathbb{R}^n : x^T L_i x < 0\}, i = 1, 2, \ldots, N.$$

It is easy to show that the system $\{L_i\}, i = 1, 2, \ldots, N$, is strictly complete if and only if

$$\bigcup_{i=1}^{N} \Omega_i = \mathbb{R}^n \backslash \{0\}.$$

Remark 2.3. As shown in [17], a sufficient condition for the strict completeness of the system $\{L_i\}$ is that there exist $\xi_i \ge 0, i = 1, 2, \ldots, N$ such that $\sum_{i=1}^N \xi_i > 0$ and $\sum_{i=1}^N \xi_i L_i < 0$. If $N = 2$ then the above condition is also necessary for the strict completeness.

Next, we introduce the following technical propositions, which will be used in the proof of the main result.

Proposition 2.4. *Let P, Q be matrices of appropriate dimensions and Q is symmetric positive definite. Then*

$$2\langle Py, x\rangle - \langle Qy, y\rangle \le \langle PQ^{-1}P^T x, x\rangle,$$

for all (x, y).

The proof of the above proposition is easily derived from completing the square:

$$0 \le \langle Q(y - Q^{-1}P^T x), y - Q^{-1}P^T x\rangle.$$

Proposition 2.5. (*Schur complement lemma*) *Given constant matrices X, Y, Z where $X = X^T, Y = Y^T, Y > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

3. Main Result

Given positive numbers $\delta, \beta, g_{ij}, f_{ij}$ we set

$$\mu = (1-\delta)^{-1}, \quad G_i = \mathrm{diag}(b_{i1}, b_{i2}, \ldots, b_{in}),$$

$$b^2 = \max\{g_{ij}^2, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\},$$

$$F_i = \mathrm{diag}(a_{i1}, a_{i2}, \ldots, a_{in}), i = 1, 2, \ldots, N,$$

$$M_i = -A_i^T P - PA_i + F_i^2 + \sum_{i=1}^N G_i^2 + 2F_i$$

$$L_i(P) = M_i + (PB_i + I)(PB_i + I)^T + e^{2\beta\tau}\mu \sum_{i=1}^N PC_i C_i^T P + 2\beta P,$$

$$\begin{aligned} \Omega_i =& \{x \in \mathbb{R}^n : \quad x^T L_i(P) x < 0\}, \\ \bar{\Omega}_1 =& \Omega_1, \quad \bar{\Omega}_i = \Omega_i \setminus \bigcup_{j=1}^{i-1} \bar{\Omega}_j, \quad i = 2, 3, \ldots, N. \end{aligned} \tag{5}$$

$$\alpha_1 = \lambda_{\min}(P), \quad \alpha_2 = \lambda_{\max}(P) + Nb^2\tau.$$

Theorem 3.1. *Given $\beta > 0$. System (3) is β-exponentially stable if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ such that the following condition holds:*

$$\exists \xi_i \ge 0, i = 1, 2, \ldots, N : \sum_{i=1}^N \xi_i > 0 : \sum_{i=1}^N \xi_i L_i(P) < 0. \tag{6}$$

The switching rule is chosen as $\sigma(x(t)) = i$ whenever $x(t) \in \bar{\Omega}_i$. Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t,\phi)\| \le \sqrt{\frac{\alpha_2}{\alpha_1}} e^{-\beta t}\|\phi\|, \quad t \in \mathbb{R}^+.$$

Proof. Consider the following Lyapunov-Krasovskii functional

$$V(t,x_t) = V_1(t,x_t) + V_2(t,x_t),$$

where

$$\begin{aligned} V_1 &= \langle Px(t), x(t)\rangle \\ V_2 &= \sum_{i=1}^{N} \int_{t-\tau(t)}^{t} e^{2\beta(s-t)} \langle g_i(x(s)), g_i(x(s))\rangle ds. \end{aligned}$$

It is easy to verify that

$$\alpha_1\|x(t)\|^2 \le V(t,x_t) \le \alpha_2\|x_t\|^2, \quad t \ge 0. \tag{7}$$

Taking derivative of $V(t,x_t)$ along trajectories of any subsystem i^{th} we have

$$\begin{aligned} \dot{V}(t,x_t) =& \langle[-A_i^T P - PA_i]x(t), x(t)\rangle + 2\langle PB_i f_i(x(t)), x(t)\rangle \\ &+ 2\langle PC_i g_i(x(t-\tau(t)), x(t)\rangle - 2\beta V_2(t,x_t) \\ &+ \sum_{i=1}^{N}\langle g_i(x(t)), g_i(x(t))\rangle - e^{-2\beta\tau(t)}(1-\dot{\tau}(t)) \times \\ &\quad \times \sum_{i=1}^{N}\langle g_i(x(t-\tau(t))), g_i(x(t-\tau(t)))\rangle. \end{aligned}$$

Using the condition $\dot{\tau}(t) \le \delta$ and Proposition 2.4 for the estimation

$$2\langle PC_i g(.), x\rangle - \frac{1-\delta}{e^{2\beta\tau}}\langle g(.), g(.)\rangle \le e^{2\beta\tau}\mu\langle PC_iC_i^T Px, x\rangle,$$

we have

$$\begin{aligned} \dot{V}(.) + 2\beta V(.) \le& \langle[-A_i^T P - PA_i + 2\beta P]x(t), x(t)\rangle \\ &+ 2\langle PB_i f_i(x(t)), x(t)\rangle + \sum_{i=1}^{N}\langle g_i(x(t)), g_i(x(t))\rangle \\ &+ e^{2\beta\tau}\mu\sum_{i=1}^{N}\langle PC_iC_i^T Px(t), x(t)\rangle. \end{aligned} \tag{8}$$

By adding and substituting into the right hand-side of the inequality (8) two items

$$2\langle f_i(x(t)), x(t)\rangle, \quad \langle f_i(x(t)), f_i(x(t))\rangle,$$

and using condition (5) and the diagonal matrices $G_i > 0, F_i > 0$ for the following estimations

$$
\begin{aligned}
-\langle f_i(x(t)), x(t)\rangle &\le \langle F_i x(t), x(t)\rangle,\\
\langle f_i(x(t)), f_i(x(t))\rangle &\le \langle F_i^2 x(t), x(t)\rangle,\\
\langle g_i(x(t)), g_i(x(t))\rangle &\le \langle G_i^2 x(t), x(t)\rangle,
\end{aligned}
$$

we have

$$
\begin{aligned}
\dot V(.) + 2\beta V(.) \le\ & \langle[-A_i^T P - PA_i]x(t), x(t)\rangle + 2\langle (PB_i + I)f_i(x(t)), x(t)\rangle\\
&+ e^{2\beta\tau}\mu \sum_{i=1}^{N} \langle PC_iC_i^T Px(t)x(t)\rangle\\
&+ \sum_{i=1}^{N} \langle G_i^2 x(t), x(t)\rangle + 2\beta\langle Px(t), x(t)\rangle\\
&+ 2\langle F_i x(t), x(t)\rangle + \langle F_i^2 x(t), x(t)\rangle - \langle f_i(x(t)), f_i(x(t))\rangle.
\end{aligned}
$$

Applying Proposition 2.4 for the estimation

$$
2\langle (PB_i + I)f_i(.), x\rangle - \langle f_i(.), f_i(.)\rangle \le \langle (PB_i + I)(PB_i + I)^T x, x\rangle
$$

we obtain that

$$
\begin{aligned}
\dot V(.) + 2\beta V(.) \le\ & \langle M_i x(t), x(t)\rangle + 2\beta\langle Px(t), x(t)\rangle\\
&+ e^{2\beta\tau}\mu \sum_{i=1}^{N} \langle PC_iC_i^T Px(t)x(t)\rangle + \langle (PB_i + I)(PB_i + I)^T x(t), x(t)\rangle\\
=\ & \langle L_i(P)x(t), x(t)\rangle.
\end{aligned}
\tag{9}
$$

By definition of the completeness and by Remark 2.3, assumption (6) implies that the system $\{L_i(P, Q, R, S)\}$ is strictly complete and we have

$$
\bigcup_{i=1}^{N} \Omega_i = \mathbb{R}^n \backslash \{0\}.
$$

Defining the sets Ω_i and $\bar\Omega_i$ by (5), we see that

$$
\bigcup_{i=1}^{N} \bar\Omega_i = \mathbb{R}^n \backslash \{0\}, \quad \bar\Omega_i \cap \bar\Omega_j = \emptyset, i \ne j.
$$

Therefore, for any $x(t) \in \mathbb{R}^n, t \ge 0$, there exists $i \in \{1, 2, \ldots, N\}$ such that $x(t) \in \bar\Omega_i$. By choosing switching rule as $\sigma(x(t)) = i$ whenever $x(t) \in \bar\Omega_i(P)$, from (9) we have

$$
\dot V(t, x_t) + 2\beta V(t, x_t) \le x^T(t) L_i(P) x(t) \le 0, \quad \forall t \ge 0.
$$

This implies that

$$V(t, x_t) \leq V(\phi)e^{-2\beta t}, t \in \mathbb{R}^+.$$

Taking the estimation (7) into account, we obtain

$$\alpha_1\|x(t,\phi)\|^2 \leq V(t, x_t) \leq V(0, x_0)e^{-2\beta t} \leq \alpha_2 e^{-2\beta t}\|\phi\|^2,$$

and hence

$$\|x(t,\phi)\| \leq \sqrt{\frac{\alpha_2}{\alpha_1}}e^{-\beta t}\|\phi\|, \quad t \in \mathbb{R}^+,$$

which concludes the proof of the theorem. ■

Remark 3.2. Note that the designed switching rule involves solving the matrix Riccati-type equation (6), which can be solved by various numerical methods given in [1, 8].

Example 3.3. Consider the switched system (1), where $\tau(t) = 3\sin^2(0.3t)$ and

$$a_{11} = a_{12} = 0.005, \quad a_{21} = a_{22} = 0.003,$$

$$b_{11} = b_{12} = 0.001, \quad b_{21} = b_{22} = 0.002,$$

$$A_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 6 \end{bmatrix}, B_1 = \begin{bmatrix} -0.05 & 0.01 \\ 0.01 & 0.05 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.0003 & 0 \\ -0.0002 & -0.0001 \end{bmatrix}, A_2 = \begin{bmatrix} 5.5 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.03 & 0 \\ -0.01 & -0.05 \end{bmatrix}, C_2 = \begin{bmatrix} -0.0005 & 0.0002 \\ 0 & 0.0004 \end{bmatrix}.$$

We have $\tau = 3, \delta = 0.9, \mu = 10$, and

$$F_1 = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.003 & 0 \\ 0 & 0.003 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.002 \end{bmatrix}.$$

The matrix inequality (6) of Theorem 3.1 is feasible with $\beta = 0.3, \xi_1 = \xi_2 = 0.5$, and

$$P = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.2 \end{bmatrix},$$

and we have

$$L_1(P) = \begin{bmatrix} 0.1031 & 0.0020 \\ 0.0020 & -0.2768 \end{bmatrix}, \quad L_2(P) = \begin{bmatrix} -0.1385 & 0.0050 \\ 0.0050 & 0.2015 \end{bmatrix}.$$

The sets Ω_1 and Ω_2 are given as follows (see Figures 1, 2):

$$\Omega_1 = \left\{x \in \mathbb{R}^2 : 1.2352x_1^2 + 0.009x_1x_2 - 1.2499x_2^2 < 0\right\},$$

$$\Omega_2 = \left\{ x \in \mathbb{R}^2 : -1.5789x_1^2 + 0.006x_1x_2 + 1.1861x_2^2 < 0 \right\}.$$

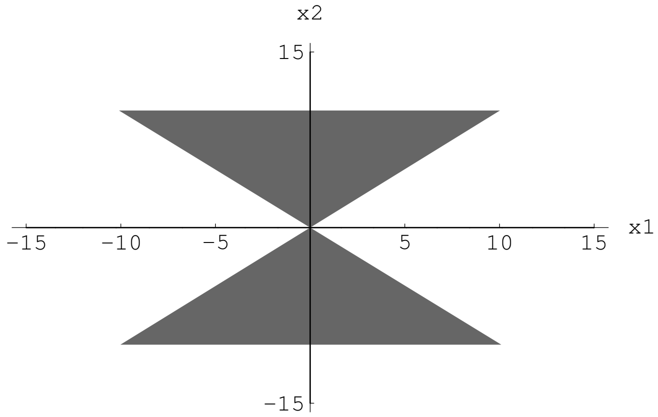

Fig. 1 The regions Ω_1

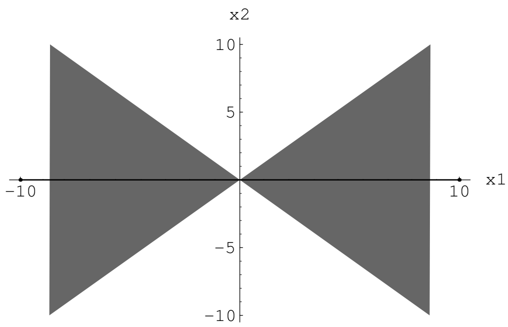

Fig. 2 The regions Ω_2

Obviously, we have $\Omega_1 \cup \Omega_2 = \mathbb{R}^2 \backslash \{(0, 0)\}$. By choosing $\xi_1 = \xi_2 = \frac{1}{2}$, we may verify that the condition of the theorem is satisfied where

$$\begin{aligned} L(P) &= \frac{1}{2}L_1(P) + \frac{1}{2}L_2(P) \\ &= \begin{bmatrix} -0.1719 & 0.0038 \\ 0.0038 & -0.0319 \end{bmatrix} < 0. \end{aligned}$$

The switching regions are given as

$$\overline{\Omega}_1 = \left\{x \in \mathbb{R}^2 : (x_1 - 1.0023x_2)(x_1 + 1.0096x_2) < 0\right\},$$
$$\overline{\Omega}_2 = \left\{x \in \mathbb{R}^2 : (x_1 - 1.0023x_2)(x_1 + 1.0096x_2) > 0\right\}.$$

According to Theorem 3.1, the system with the switching rule $\sigma(x(t)) = i$ if $x(t) \in \overline{\Omega}_i$ is $0.3-$ exponentially stable. Moreover, by straightforward calculation, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t,\phi)\| \le 1.118e^{-0.3t}\|\phi\|, \quad t \in \mathbb{R}^+.$$

4. Conclusion

This paper has proposed a switching design for the exponential stability of switched neural networks with time-varying delay and various activation functions. The stability condition is derived in terms of the solution of Riccati-type equations. The approach allows for the use of efficient techniques for computation of the two bounds that characterize the exponential stability rate of the solution.

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