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Eigenvalue Approach to Two-Temperature Magneto-Thermoelasticity

Nantu Sarkar and Abhijit Lahiri

Department of Mathematics, Jadavpur University, Kolkata-700032, India

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Abstract. A generalized thermoelastic theory, in the context of L-S theory, is used to investigate the two-temperature magneto-thermoelastic one-dimensional problem for a perfect conducting infinite medium whose surface is subjected to a thermal shock and is either considered as (i) traction-free or (ii) laid on a rigid foundation. The one-dimensional generalized magneto-thermoelastic coupled governing equations are written into a vector-matrix differential equation by using Laplace transform techniques and then solved by eigenvalue approach. The inversion of the transforms solution is carried out numerically in the space-time domain using Bellman method and illustrated graphically in two different cases. The effects of applied magnetic field and the two-temperature parameter on the field variables are studied.

2000 Mathematics Subject Classification. 74F15.

Key words. Generalized thermoelasticity, magneto-thermoelasticity, two-temperature parameter, conducting medium, Laplace transform, eigenvalue.

1. Introduction

The classical theory of uncoupled thermoelasticity predicts two phenomena not compatible with physical observation. Firstly, the heat conduction equation of this theory does not contain any elastic terms contrary to the fact that the elastic changes produced heat effects. Secondly, the heat conduction equation is of parabolic type predicting infinite speed of propagation for heat waves. To overcome the first shortcoming, Biot [3] introduced the coupled thermoelasticity. In this theory the governing equations are coupled eliminating the first paradox of classical theory. To eliminate the second shortcoming the theory of generalized thermoelasticity were developed. The generalized thermoelasticity theories admit so-called second sound effects i.e., which predicts finite velocity of propagation for heat flux. At present there are two different theories of generalized thermoelasticity, (i) Lord and Shulman [13] (L-S theory); and (ii) Green and Lindsay [9] (G-L theory).

The L-S model itself based on a modified Fourier's law and obtained wave type heat conduction equation. The L-S model contains the heat flux vector and its time derivative. In this case there is a new constant that acts as a time relaxation parameter. In G-L theory there are two relaxation time parameters and modified not only heat conduction equation but also all the equations of coupled theory without violating Fourier's law. Both the theories are structurally different and one cannot be obtained as particular of other.

Now a days in geophysics, plasma physics, nuclear fields and related topics magneto field and stress, strain in thermoelastic materials has a very important role. In the field of magneto-thermoelasticity many studies have been conducted by different researchers on either L-S model or G-L model. Sharma and Chand [15], Chandrasekharaiah et al. [4], and Choudharuy et al. [14] studied one dimensional magneto-thermoelasticity problems. Other relevant papers studied by different researchers are Sherief and Ezzet [16, 17], Sherief and Helmy [18], Ezzet and Othman [6, 7], Tianhu and Shirong [10], Tuphole [20], and Ezzet and Bary [8].

Most of the thermoelasticity, magneto-thermoelasticity (generalized or coupled) problems have been solved by using potential functions. This method is not always suitable as discussed by Sherief [1] and Sherief and Anwar [19]. These may be summarized by (i) the boundary and initial conditions for physical problems are directly related to the physical quantities under consideration and not to the potential function and (ii) the solution of the physical problem in natural variables is convergent while other potential function representations is not convergent always.

The alternative to the potential function approach are - (i) State-Space approach: This method is essentially an expansion in a series in terms of the coefficient matrix of the field variables in ascending powers and applying Caley-Hamilton theorem, which requires extensive algebra, and (ii) Eigenvalue approach: This method reduces the problem on vector-matrix differential equation to an algebraic eigenvalue problems and the solutions for the field variables are achieved by determining the eigenvalues and the corresponding eigenvectors of the coefficient matrix. In eigenvalue approach the physical quantities are directly involved in the formulation of the problem and as such the boundary and initial conditions can be applied directly. Body forces and/or heat sources are also accommodated in both the theories, see Das et al. [5], Lahiri et al. [12], Kar et al. [11].

In this work, we consider a one-dimensional two-temperature magneto-thermoelastic problem for a perfect conducting infinite medium in the context of generalized thermoelastic theory, whose surface is subjected to a thermal shock and is either considered as (i) traction-free or (ii) laid on a rigid foundation. The main objects of this present paper is to study the above problem based on the L-S theory with the help of the eigenvalue approach proposed by Das et al. [5]. The inversion of the Laplace transform solutions are carried out numerically using Bellman method [2] and illustrated graphically in two different cases. The effects of applied magnetic field and the two-temperature parameter on stress, temperature, displacement, induced magnetic and electric field are studied.

2. Formulation of the problem

In our present work, we consider a thermo-viscoelastic infinite medium of perfect conductivity permeated by an initial magnetic field $\mathbf{H}=(0, H_0, 0)$. This produces an induced magnetic field $\mathbf{h}=(0, h, 0)$ and induced electric field $\mathbf{E}=(0, 0, E)$ which satisfy the linear equations of electromagnetism and are valid for slowly moving media of perfect conductivity [8]

$$\operatorname{curl}\mathbf{h} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

$$\operatorname{curl}\mathbf{E} = -\mu_0 \frac{\partial \mathbf{h}}{\partial t}, \tag{2}$$

$$\mathbf{E} = -\mu_0 (\frac{\partial \mathbf{u}}{\partial t} \wedge \mathbf{H}), \tag{3}$$

$$\operatorname{div}\mathbf{h} = 0. \tag{4}$$

Considering Lorentz force, the equation of motion in terms of displacements and heat-conduction equation are

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + \mu_0 (\mathbf{J} \wedge \mathbf{H})_i, \tag{5}$$

$$K\Phi_{,ii} = \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) [\rho C_E T + \gamma t_0 e]. \tag{6}$$

The relation between the conductive temperature Φ and the thermodynamic temperature θ is given by [8]

$$\Phi - T = a\Phi_{,ii}, \tag{7}$$

where $a > 0$ is the temperature discrepancy.

The stress components are given by

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma (T - T_0), \tag{8}$$

where the strain components are given by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{9}$$

In the above equations a comma denotes space derivatives and the summation convention is used. We now consider a homogeneous, isotropic thermoelastic conducting solid occupying the half space $x \geq 0$, which is initially at rest. The displacement components are of the form

$$u_x = u(x,t) \; u_y = u_z = 0$$

and hence

$$e = e_{xx} = \frac{\partial u}{\partial x}. \tag{10}$$

The heat conduction equation (6) becomes

$$K\frac{\partial^2 \Phi}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau\frac{\partial^2}{\partial t^2}\right)[\rho C_E T + \gamma T_0 e], \tag{11}$$

and the stress components will be of the form

$$\sigma = \sigma_{xx} = (\lambda + 2\mu)e - \gamma T. \tag{12}$$

In this problem, a constant magnetic field with components $(0, H_0, 0)$ is permeating the medium.One component of induced magnetic field **h** is along y-direction, while induced electric field **E** will have one component along z-direction. Thus Eqs. (1)–(3) reduce to

$$J = \left(\frac{\partial h}{\partial x} - \varepsilon_0\frac{\partial E}{\partial t}\right), \tag{13}$$

$$h = -H_0 e, \tag{14}$$

$$E = -\mu_0 H_0 \frac{\partial u}{\partial t}. \tag{15}$$

Using equations (14) and (15) into (13) and using equation (1), the displacement equation (5) can be written as

$$\frac{\partial^2 \sigma}{\partial x^2} + \mu_0 H_0^2 \frac{\partial^2 e}{\partial x^2} = \rho\alpha\frac{\partial^2 e}{\partial t^2}, \tag{16}$$

where $\alpha = 1 + \frac{\alpha_0^2}{c^2}$, $\alpha_0^2 = \frac{\mu_0 H_0^2}{\rho}$, $c^2 = \frac{1}{\varepsilon_0\mu_0}$.
Using the non-dimensional variables defined as follows

$$x^* = c_0\eta_0 x,\; t^* = c_0^2\eta_0 t,\; u^* = c_0\eta_0 u,\; \theta^* = \frac{\gamma(T - T_0)}{\rho c_0^2},\; \Phi^* = \frac{\gamma(\Phi - \Phi_0)}{\rho c_0^2}$$

$$\sigma^* = \frac{\sigma}{\rho c_0^2},\; \eta_0 = \frac{\rho C_E}{K},\; \varepsilon = \frac{\delta_0\gamma}{\rho C_E},\; h^* = \frac{h}{H_0}\; E^* = \frac{E}{\mu_0 H_0 c_0},\; c_0^2 = \frac{\lambda + 2\mu}{\rho}\;;$$

the basic equations can be written as follows (dropping the asterisks for convenience)

$$h = -e, \tag{17}$$

$$E = -\frac{\partial u}{\partial t}, \tag{18}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = (\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2})[\theta + \varepsilon e], \tag{19}$$

$$e = \sigma + \theta, \tag{20}$$

$$\frac{\partial^2 \sigma}{\partial x^2} + \beta \frac{\partial^2 e}{\partial x^2} = \alpha \frac{\partial^2 e}{\partial t^2}, \tag{21}$$

$$\Phi - \theta = \beta_0 \frac{\partial^2 \Phi}{\partial x^2}, \tag{22}$$

where $\beta = \frac{\alpha_0^2}{c_0^2}$, $\beta_0 = a c_0^2 \eta_0^2$.

3. Formulation of the vector-matrix differential equation

We now apply the Laplace transform defined by

$$\bar{f}(x,s) = \int_0^\infty f(x,t) e^{-st} dt$$

(where s is the transform parameter such that Re(s)> 0) to both sides of equations (17)-(22) and assuming that all the initial state functions along with their derivatives with respect to t are equal to zero, we obtain

$$\bar{h} = -\bar{e}, \tag{23}$$

$$\bar{E} = -s\bar{u}, \tag{24}$$

$$\frac{d^2 \bar{\Phi}}{dx^2} = (s + \tau s^2)[\bar{\theta} + \varepsilon \bar{e}], \tag{25}$$

$$\bar{e} = \bar{\sigma} + \bar{\theta}, \tag{26}$$

$$\frac{d^2 \bar{\sigma}}{dx^2} + \beta \frac{d^2 \bar{e}}{dx^2} = \alpha s^2 \bar{e}, \tag{27}$$

$$\bar{\Phi} = \bar{\theta} + \beta_0 \frac{d^2 \bar{\Phi}}{dx^2}. \tag{28}$$

Eliminating $\bar{e}$ and $\bar{\theta}$ from equations (25)-(28) and after simplifying we get

$$\frac{d^2 \bar{\Phi}}{dx^2} = C_1 \bar{\Phi} + C_2 \bar{\sigma}, \tag{29}$$

and

$$\frac{d^2 \bar{\sigma}}{dx^2} = D_1 \bar{\Phi} + D_2 \bar{\sigma}, \tag{30}$$

where

$$C_1 = \frac{(1+\varepsilon)(s+\tau s^2)}{[1+\beta_0(1+\varepsilon)(s+\tau s^2)]}\ ,\ C_2 = \frac{\varepsilon(s+\tau s^2)}{[1+\beta_0(1+\varepsilon)(s+\tau s^2)]}, \tag{31}$$

and

$$D_1 = \frac{(1-\beta_0 C_1)(\alpha s^2 - \beta C_1)}{[1+\beta(1-\beta_0 C_2)]}\ ,\ D_2 = \frac{(1-\beta_0 C_2)(\alpha s^2 - \beta C_2)}{[1+\beta(1-\beta_0 C_2)]}. \tag{32}$$

As in [5], the resulting equations (29) and (30) can be written in the form of a *vector-matrix differential equation* as follows

$$\frac{d\tilde{V}}{dx} = \tilde{A}\tilde{V}, \tag{33}$$

where

$$\tilde{V}(x,s) = \left[\overline{\Phi}\ \overline{\sigma}\ \overline{\Phi}'\ \overline{\sigma}'\right]^T, \tag{34}$$

and

$$\tilde{A}(x,s) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ C_1 & C_2 & 0 & 0 \\ D_1 & D_2 & 0 & 0 \end{bmatrix}. \tag{35}$$

4. Solution of the vector-matrix differential equation

Following the solution methodology through eigenvalue approach [5], we now proceed to solve the vector-matrix differential equation (33). The characteristic equation of the matrix $\tilde{A}$ is

$$k^4 - (C_1 + D_2)k^2 + (C_1 D_2 - C_2 D_1) = 0.$$

Let k_1^2 and k_2^2 be the roots of the above characteristic equation with positive real parts. Then all the four roots of the above characteristic equation which are also the eigenvalue of the matrix $\tilde{A}$ are of the form

$$k = \pm k_1 \text{ and } \pm k_2,$$

where $k_1^2 + k_2^2 = C_1 + D_2,\ k_1^2 k_2^2 = C_1 D_2 - C_2 D_1$.

The right eigenvector, $\tilde{X} = [X_1,\ X_2,\ X_3,\ X_4]^T$, say, corresponding to the eigenvalue k can be written as

$$\tilde{X} = \left[(k^2 - D_2),\ D_1,\ k(k^2 - D_2),\ kD_1\right]^T. \tag{36}$$

From (36) we can easily calculate the eigenvector $\tilde{X}_i$, corresponding to the eigenvalue k_i, $i = 1,2,3,4$. For our further reference we shall use the following notations

$$\tilde{X}_1 = [\tilde{X}]_{k=k_1},\ \tilde{X}_2 = [\tilde{X}]_{k=-k_1},\ \tilde{X}_3 = [\tilde{X}]_{k=k_2},\ \tilde{X}_4 = [\tilde{X}]_{k=-k_2}. \tag{37}$$

The solution of the equation (33) may be written as

$$\tilde{V}(x,p) = A_1\tilde{X}_2 e^{-k_1 x} + A_2\tilde{X}_4 e^{-k_2 x}\ ,\ x \geq 0 \tag{38}$$

where the terms containing exponentials of growing nature in the space variables x has been discard due to the regularity condition of the solution at infinity and $A_1,\ A_2$ are constants to be determined from the boundary condition of the problem.

Thus the field variables can be written from (38), for $x \geq 0$ as

$$\bar{\Phi}(x,s) = A_1(k_1^2 - M_2)e^{-k_1 x} + A_2(k_2^2 - M_2)e^{-k_2 x}, \tag{39}$$

$$\bar{\sigma}(x,s) = A_1 M_1 e^{-k_1 x} + A_2 M_1 e^{-k_2 x}, \tag{40}$$

where we use $D_j = M_j\ (j = 1,\ 2)$. Using equation (39) in (28), we obtain

$$\bar{\theta}(x,s) = A_1(k_1^2 - M_2)(1 - \beta_0 k_1^2)e^{-k_1 x} + A_2(k_2^2 - M_2)(1 - \beta_0 k_2^2)e^{-k_2 x}. \tag{41}$$

Again using equations (40) and (41) in (26) and after simplifying, we get the strain component as

$$\bar{e}(x,s) = A_1 P_1 e^{-k_1 x} + A_2 P_2 e^{-k_2 x}, \tag{42}$$

where

$$P_1 = [M_1 + (k_1^2 - M_2)(1 - \beta_0 k_1^2)], \tag{43}$$

$$P_2 = [M_1 + (k_2^2 - M_2)(1 - \beta_0 k_2^2)]. \tag{44}$$

From equation (27), the displacement component takes the form

$$\bar{u}(x,s) = \frac{1}{\alpha s^2}\left[\frac{d\bar{\sigma}}{dx} + \beta\frac{d\bar{e}}{dx}\right]. \tag{45}$$

Using equations (40) and (42) in the equation (45), the displacement component can be calculated as

$$\bar{u}(x,s) = \frac{-1}{\alpha s^2}[A_1 k_1(M_1 + \beta P_1)e^{-k_1 x} + A_2 k_2(M_1 + \beta P_2)e^{-k_2 x}]. \tag{46}$$

The induced magnetic and electric field are obtained by using the equations (42) and (46) to equations (23) and (24) respectively as

$$\bar{h}(x,s) = -A_1 P_1 e^{-k_1 x} - A_2 P_2 e^{-k_2 x}, \tag{47}$$

$$\bar{E}(x,s) = \frac{1}{\alpha s}[A_1 k_1(M_1 + \beta P_1)e^{-k_1 x} + A_2 k_2(M_1 + \beta P_2)e^{-k_2 x}]. \tag{48}$$

Boundary conditions

We consider a homogeneous elastic medium of perfect conductivity occupying the region $x \geq 0$, with quiescent initial state and boundary conditions in the following form

Case-I:

(i) thermal boundary condition: the surface $x = 0$ is subjected to a thermal shock in the form

$$\Phi(0,t) = \Phi_0 e^{-\omega t} \Rightarrow L[\Phi(0,t)] = L[\Phi_0 e^{-\omega t}] \Rightarrow \bar{\Phi}(0,s) = \frac{\Phi_0}{s+\omega}; \tag{49}$$

(i) mechanical boundary condition: the surface $x = 0$ is laid on a rigid foundation; that is

$$u(0,t) = 0 \Rightarrow L[u(0,t)] = 0 \Rightarrow \bar{u}(0,s) = 0, \tag{50}$$

where L stands for Laplace transformation.

Using equations (49) and (50) in (39) and (46) we obtain A_1 and A_2 as:

$$A_1 = \frac{\Phi_0 k_2[M_1 + \beta P_2]}{(s+\omega)(k_1^2 - M_2)k_2[M_1 + \beta P_2] - (s+\omega)(k_2^2 - M_2)k_1[M_1 + \beta P_1]}, \tag{51}$$

$$A_2 = -\frac{k_1[M_1 + \beta P_1]}{k_2[M_1 + \beta P_2]} A_1. \tag{52}$$

Case-II:

(i) thermal boundary condition: the surface $x = 0$ is subjected to a thermal shock in the form

$$\Phi(0,t) = \Phi_0 H(t) \Rightarrow L[\Phi(0,t)] = L[\Phi_0 H(t)] \Rightarrow \bar{\Phi}(0,s) = \frac{\Phi_0}{s}; \tag{53}$$

(i) mechanical boundary condition: the surface $x = 0$ is taken to be traction free; that is,

$$\sigma(0,t) + T_{11}(0,t) - T_{11}^0(0,t) = 0, \tag{54}$$

where $T_{11}^0(0,t)$ is the Maxwell stress tensor in the vacuum.

Since induced electric field E and induced magnetic field h are continuous across the bounding plane, i.e. $E(0,t) = E^0(0,t)$ and $h(0,t) = h^0(0,t), t > 0$, where E^0 and h^0 are the components of the induced electric and magnetic field in free space and the numerical value of the relative permeability is almost equal to unity, it follows that $T_{11}(0,t) = T_{11}^0(0,t)$ and equation (54) reduces to

$$\sigma(0,t) = 0 \Rightarrow L[\sigma(0,t)] = 0 \Rightarrow \bar{\sigma}(0,s) = 0. \tag{55}$$

Using equations (53) and (55) in (39) and (40) respectively we obtain A_1 and

A_2 as

$$A_1 = \frac{\Phi_0}{s(k_1^2 - k_2^2}\ ,\ A_2 = -\frac{\Phi_0}{s(k_1^2 - k_2^2}. \tag{56}$$

5. Numerical results and discussion

The Laplace inversion of the expressions given in (39)-(42) and in (46)-(48) for the field variables conductive and thermodynamic temperature, stress, displacement, induced magnetic and electric field respectively in space-time domain are very complex and we prefer to develop an efficient computer programme for the inversion of these integral transforms. For this inversion of Laplace transform we follow the method of Bellman [2] and choose seven values of the time $t = t_i : i = 1, 2, 3, 4, 5, 6, 7$ as the time range at which the above field variables are to be determined where t_i are the roots of the Legendre polynomial of degree seven.

With an aim to illustrate the problem, we will present some numerical results graphically. For this purpose, numerical computation is carried out for two cases for the copper material, which has the following data [8]

Values of the constants:

$K = 386\ N/Ks$	$\alpha_t = 1.78(10)^{-5}\ K^{-1}$	$C_E = 383.1\ m^2/K$
$\eta_0 = 8886.73m/s^2$	$c_0 = 4.158(10)^3$	$\varepsilon_0 = (10)^{-9}/(36\pi)C^2/Nm^2$
$\mu_0 = 4(10){-}7$	$\beta = 0.1, \beta_0 = 0.1$	$H_0 = 1.0, \alpha = 1.5$
$\mu = 3.86(10)^{10}\ N/m^2$	$\lambda = 7.76(10)^{10}\ N/m^2$	$\rho = 8954\ kg/m^3$
$\tau = 0.02\ s$	$T_0 = 293\ \mathbf{K}$	$\varepsilon = 0.0168$

In order to study the characteristic of all the field variables we have drawn several graphs for different values of the space variable x and at times $t_1 = 0.025775, t_2 = 0.138382, t_3 = 0.352509, t_4 = 0.693147, t_5 = 1.21376, t_6 = 2.04612, t_7 = 3.67119$ taking different values of α and the two-temperature parameter β_0 in both the cases using *cubic spline formulation* in *MATLAB* software. In all figures for both cases, we observe the following results:

6. Concluding remarks

Case-I

(a) Figs. 1-3 exhibit the variation of conductive temperature Φ, stress σ and thermodynamic temperature θ with time t. We observe that: i) For fixed value of x and β_0 the absolute value of Φ, σ and θ gradually decreases as t increases. ii) The absolute value of Φ, σ and θ decreases for fixed β_0 and t as x increases. iii) In general, the absolute value of Φ, σ and θ for fixed t and x, when $\beta_0 = 0$ are greater than $\beta = 0.1$.

(b) Figs. 4-6 exhibit the variation of displacement u, induced magnetic field h and induced electric field E with time t. We observe that: i) For fixed value of x and α the amplitudes of u, h and E gradually decreases for $0 < t \leq 1.5$ and then increases for $1.5 < t \leq 3.67$ with greater wave length as t increases. ii) We see that u and h attain their maximum absolute value at $t = 0.25$ when $x = 0.2$ and $\alpha = 1.0$ while E attains its maximum (absolute) value at $t = 0.578$ when $x = 0.1$ and $\alpha = 1.0$. iii) The curves of u and E are smoother in case $x = 2$ than in case $x = 1$ while the curves of h are smoother in case $\alpha = 1.5$ than in case $\alpha = 1.01$.

Case-II

(a) Fig. 7 shows that: i) For fixed t, α and β_0, the numerical value of conductive temperature Φ gradually decreases as x increases. ii) The curves are smoother in case $x = 0.1$ than in case $x = 0.2$ and $x = 0.3$. iii) At time $t = t_1$, Φ attains its maximum value for $x = 0.1$.

(b) Fig. 8 shows that: i) The value of stress σ gradually increases in some location $0.05 \leq x \leq 0.1$ and then decreases gradually for $0.1 < x \leq 0.17$.Significant changes are not shown for $x \geq 0.18$. ii) For fixed x, β_0 and α the value of σ is maximum at time $t = t_3$. iii) The stress is compressive for $0 < x \leq 0.03$ for fixed time t, β_0 and α.

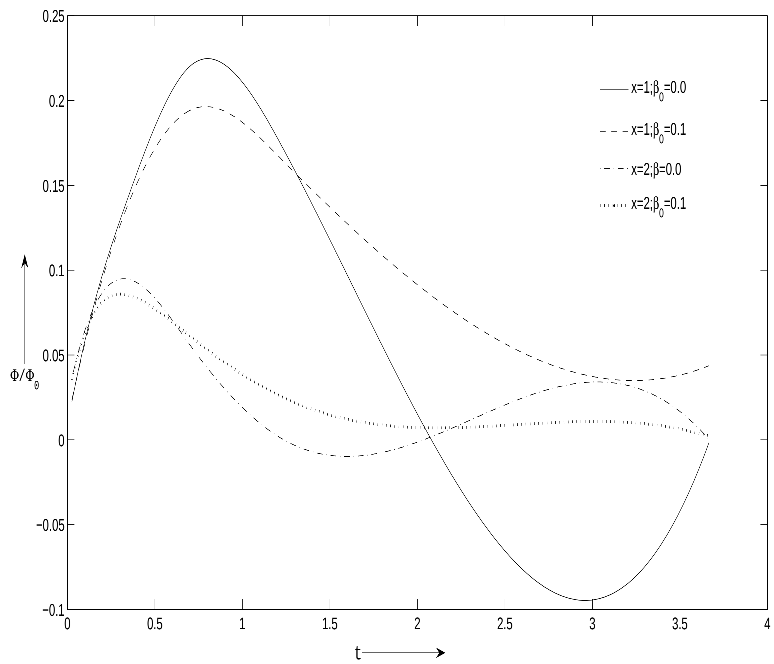

Fig. 1 The conductive temperature distribution at $\alpha = 1.5$ for case I.

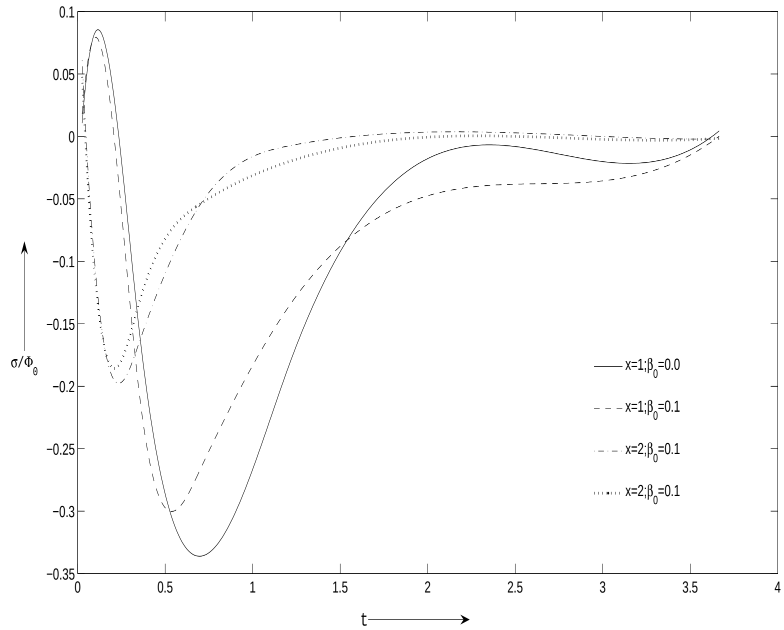

Fig. 2 The stress distribution at $\alpha = 1.5$ for case I.

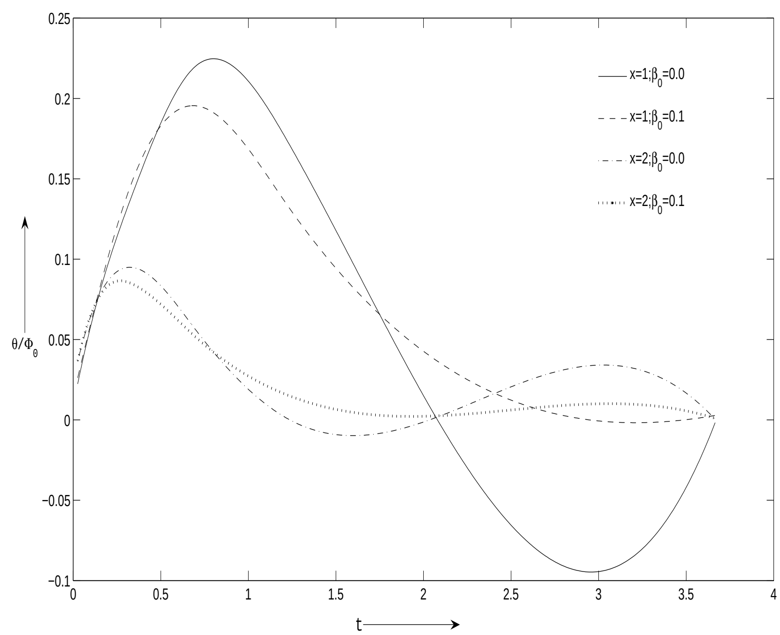

Fig. 3 The thermodynamic temperature distribution at $\alpha = 1.5$ for case I.

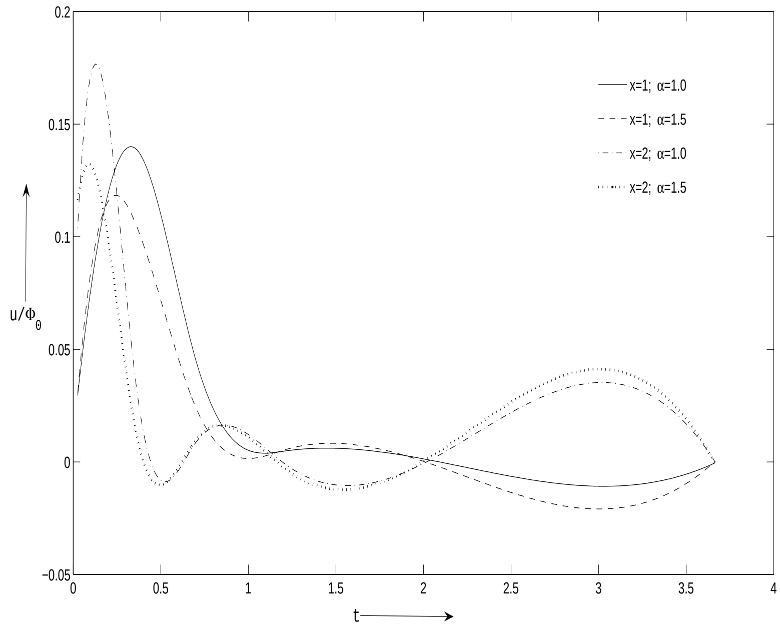

Fig. 4 The displacement distribution at $\beta_0 = 0.1$ for case I.

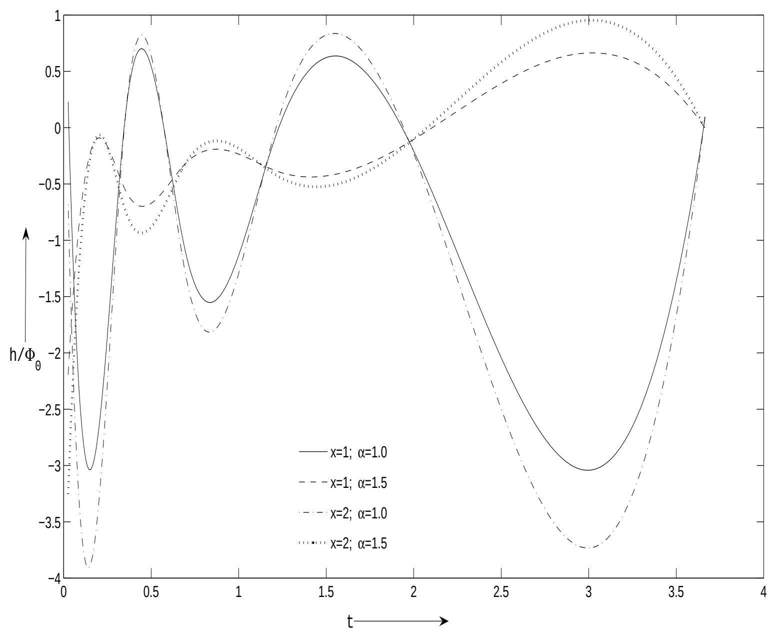

Fig. 5 The induced magnetic field distribution at $\beta_0 = 0.1$ for case I.

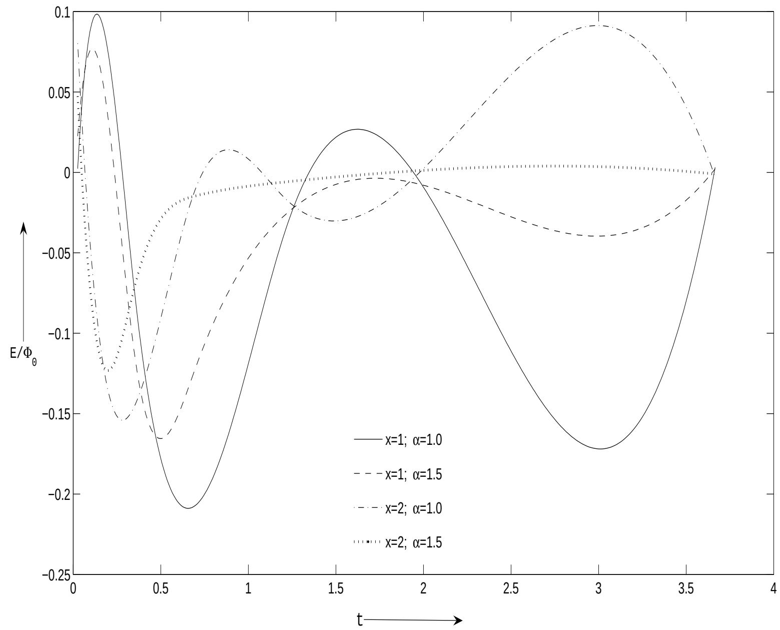

Fig. 6 The induced electric field distribution at $\beta_0 = 0.1$ for case I.

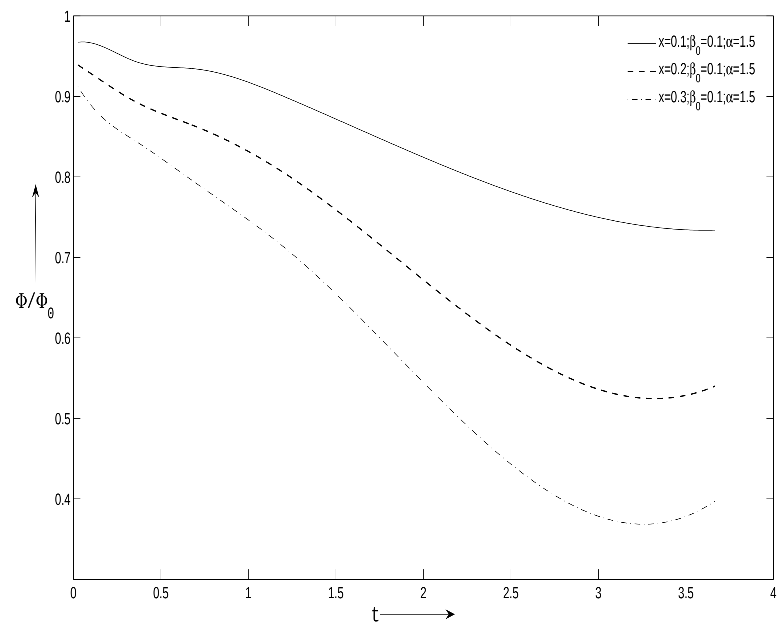

Fig. 7 The conductive temperature distribution for case II.

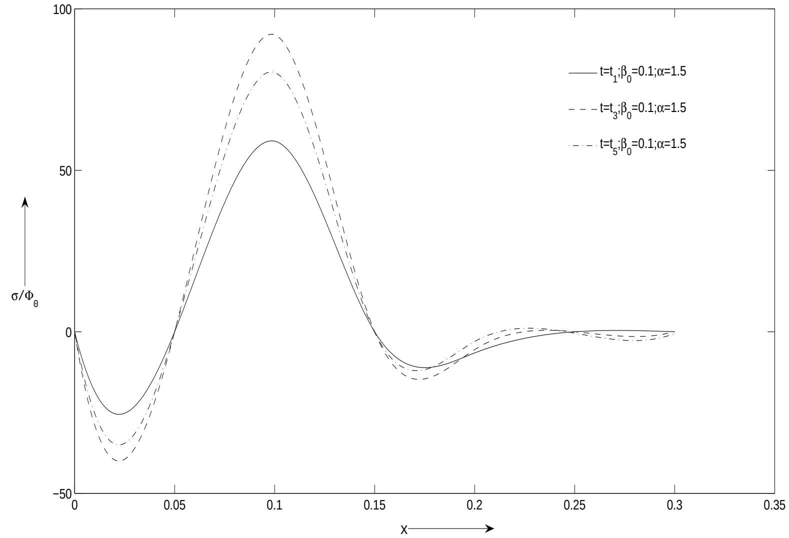

Fig. 8 The thermodynamic temperature distribution for case II.

(c) Fig. 9 shows that: i) For fixed x, α and β_0, the numerical value of thermodynamic temperature θ gradually decreases as time t increases its value is maximum near $t = t_1$ when $x = 1$.

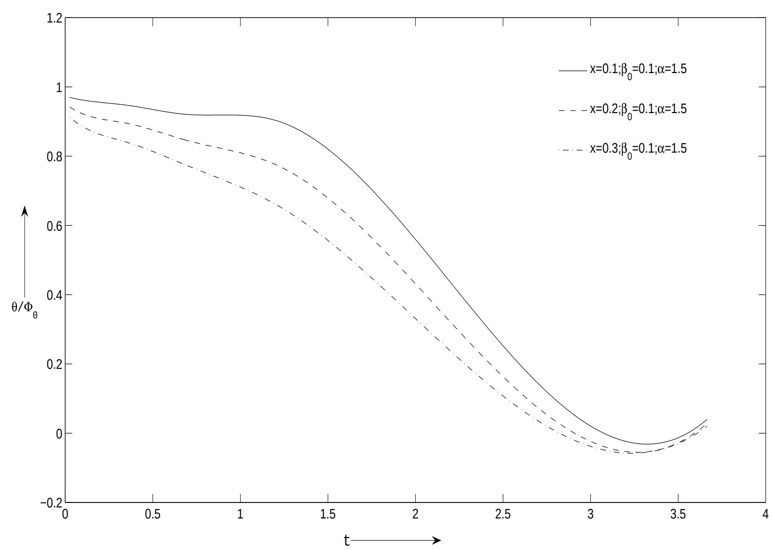

Fig. 9 The stress distribution for case II.

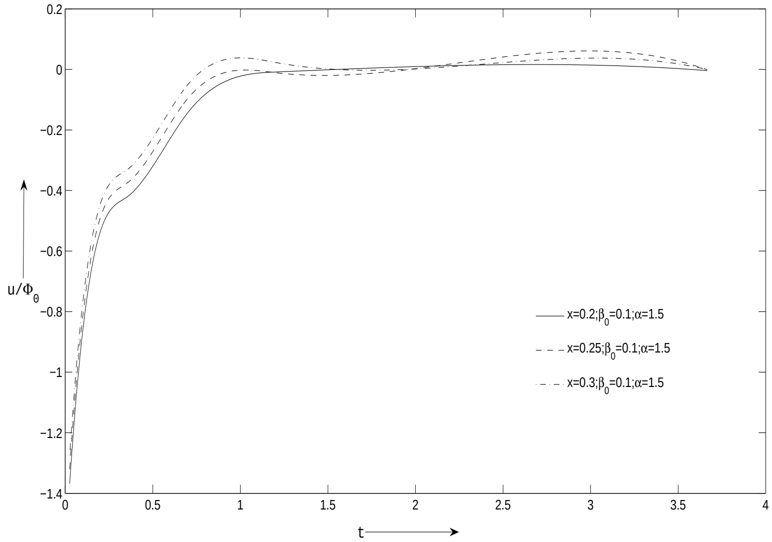

Fig. 10 The displacement distribution for case II.

(d) Fig. 10 exhibit that: i) The absolute value of u changes rapidly in $0 \leq t \leq 1$ and nothing significant changes shown for $t \geq 0.18$ for fixed x, α and β_0 and attains its maximum absolute value near $t = t_1$ for $x = 0.2$.

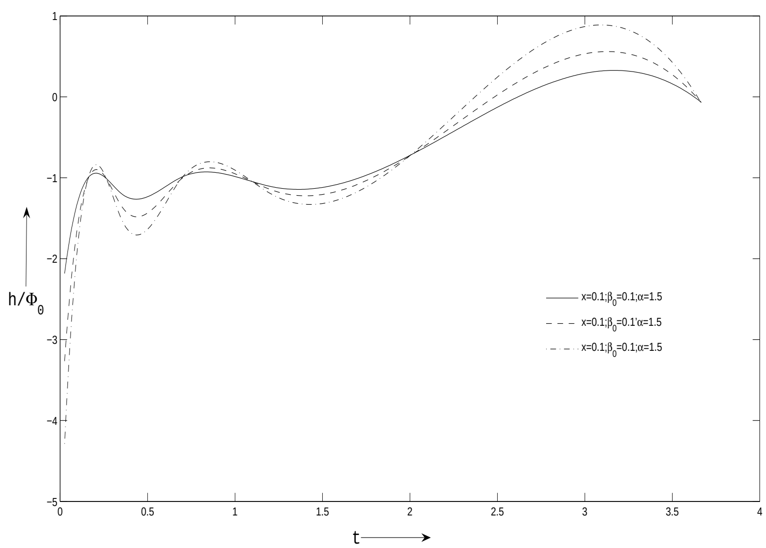

Fig. 11 The induced magnetic field distribution for case II.

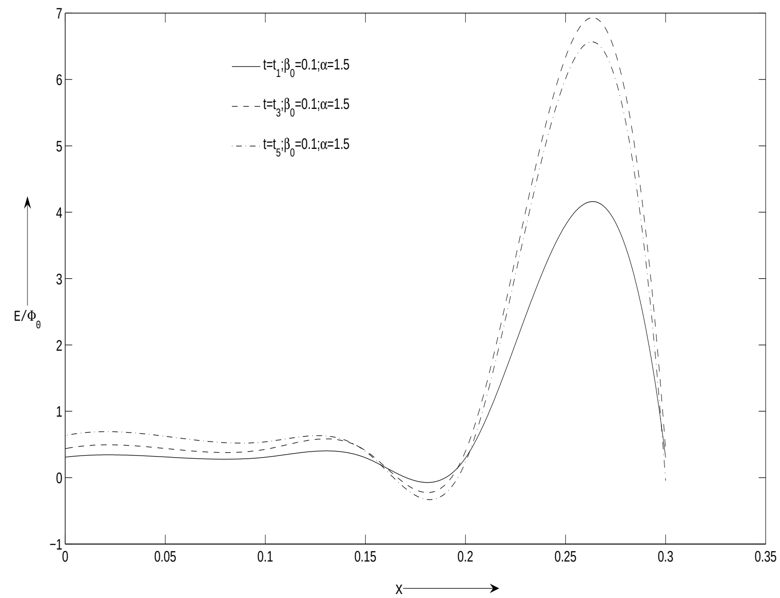

Fig. 12 The induced electric field distribution for case II.

(e) From Fig. 11 we see that the numerical value of induced magnetic field h rapidly changes in $0 \leq t \leq 0.25$ for fixed values of x, α and β_0 then its changes are not significant and it attains maximum absolute value near $t = t_1$ for $x = 0.3$.

(f) From Fig. 12 it is observe that: i) There is nothing significant changes of the induced electric field E in $0 \geq x \geq 0.16$ and it increases rapidly in $0.2 \geq x \geq 0.25$ and then decreases rapidly in $0.26 \geq x \geq 0.3$. ii) the numerical value of E is maximum when $t = t_1$ near $x = 0.25$. iii) The curves are smoother in case $t = t_1$ than in case $t = t_3$ and $t = t_5$.

7. Nomenclature

$\lambda, \mu \rightarrow$ Lamè's constants.
$C_E, \rho \rightarrow$ Specific heat at constant strain and Density respectively.
$t, \tau \rightarrow$ Time and Relaxation Time Parameter respectively.
$u_i, \sigma_{ij} \rightarrow$ Components of displacement vector and Stress respectively.
$e =\text{div}\mathbf{u} \rightarrow$ Cubical dilation.
$\mathbf{H}, \mathbf{E} \rightarrow$ Magnetic and Electric field intensity vector respectively.
$H_0 \rightarrow$ Constant component of the magnetic field.
$\mathbf{J} \rightarrow$ Conduction current density vector.
$\theta = T - T_0 \rightarrow$ Thermodynamic temperature where T_0 is the Reference temperature.

$\Phi \to$ Conductive temperature.
$K \to$ Coefficient of thermal conductivity.
$\mu_0, \varepsilon_0 \to$ Magnetic and Electric permeability respectively.
$\delta_{ij} \to$ Kronecker delta
$\delta_0 = \frac{\gamma T_0}{\rho c^2} \to$ Dimensionless constant for adjustment for the reference temperature.
$\beta_0 \to$ The dimensionless temperature discrepancy.
$\varepsilon = \frac{\delta_0 \gamma}{\rho C_E} \to$ Thermal coupling parameter.
$\gamma = (3\lambda + 2\mu)\alpha_T$; $\alpha_T \to$ Coefficient of linear thermal expansion at constant strain.
$c_0^2 = \frac{\lambda+2\mu}{\rho} \to$ Speed of propagation of isothermal elastic waves.
$\alpha_0^2 = \frac{\mu_0 H_0^2}{\rho} \to$ Alfven velocity.
$c^2 = \frac{1}{\mu_0 \varepsilon_0} \to$ Speed of light.

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