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Intuitionistic Q-Fuzzy Ideals of Near-Rings

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Abstract. In this paper we introduce and study the concept of intuitionistic Q-fuzzy ideals of near-rings. Also we derive some properties of intuitionistic Q-fuzzy ideals of near-rings.

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Key words. Fuzzy set, intuitionistic fuzzy set, Q-fuzzy set, intuitionistic Q-fuzzy set, fuzzy ideal over Q, intuitionistic Q-fuzzy ideal, near-ring.

1. Introduction

Near-Ring is a generalized structure of a ring. The theory of fuzzy sets was introduced by Zadeh [16]. The fuzzy set theory has been developed in many directions by the research scholars. Rosenfeld [15] first introduced the fuzzification of the algebraic structures and defined fuzzy subgroups.

The intuitionistic fuzzy sets (IFSs) are substantial extensions of the ordinary fuzzy sets. IFSs are objects having degrees of membership and non-membership such that their sum is less than or equal to 1. The most important property of IFSs not shared by the fuzzy sets is that model-like operators can be defined over IFSs. The IFSs have essentially higher describing possibilities than fuzzy sets.

The notion of intuitionistic fuzzy sets was introduced by Atanasov [3] as a generalization of the notion of fuzzy sets. Biswas [4] applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. The notion of an intuitionistic fuzzy ideal of a near-ring is given by Jun, Kim and Yon [10]. Also Cho and Jun [5] introduced the notion of intuitionistic fuzzy R-subgroups in a near-ring and related properties are investigated. Kazanci, Yamak and Yilmaz [11] introduced intuitionistic Q-fuzzy R-subgroups of near-rings. Kim [13] introduced sensible fuzzy R-subgroups of near-rings with respect to s-norm. Kim [12] introduced itenuitionistic Q-fuzzy semiprime ideals of semigroups. Davvaz and Dudek [6] defined intuitionistic fuzzy H_v-ideals of a ring and derived their properties. Davvaz, Dudek and Jun [7] introduced intuitionistic fuzzification of H_v-submodules in a H_v-module. In this paper we introduce the notion of intuitionistic Q-fuzzy ideals of near-rings and investigate some related properties.

2. Definitions and preliminaries

We recall some definitions for the sake of completeness. Throughout this paper Q denotes any non-empty set and R is a near-ring.

Definition 2.1. [9, 14] By a near-ring we mean a non-empty set R with two binary operations " $+$ " and " $\cdot$ " satisfying the following axioms:

(i) $(R, +)$ is a group;

(ii) $(R, \cdot)$ is a semi-group;

(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring". We denote xy instead of $x \cdot y$. Note that $x0 = 0$ and $x(-y) = -xy$, but $0x \neq 0$ for $x, y \in R$.

Definition 2.2. [1, 2, 8] An ideal I of a near-ring R is a subset of R such that

(i) $(I, +)$ is a normal subgroup of $(R, +)$;

(ii) $RI \subseteq I$;

(iii) $(r + i)s - rs \in I$ for all $i \in I$ and $r, s \in R$.

Note that if I satisfies (i) and (ii) then it is called a left ideal of R.

If I satisfies (i) and (iii) then it is called a right ideal of R.

Definition 2.3. [11] A function $\mu : R \times Q \to [0, 1]$ is called a Q-fuzzy set.

Definition 2.4. [11] Let μ be a Q-fuzzy set in R and $t \in [0, 1]$, then an upper t-level cut of μ is defined by

$$U(\mu; t) = \{x \in R \mid \mu(x, q) \geq t, q \in Q\}.$$

Definition 2.5. [11] Let μ be a Q-fuzzy set in R and $t \in [0,1]$, then a lower t-level cut of μ is defined by

$$L(\mu; t) = \{x \in R \mid \mu(x,q) \leq t, q \in Q\}.$$

Definition 2.6. [11] An intuitionistic Q-fuzzy set A is an object having the form

$$A = \{((x,q), \mu_A(x,q), \lambda_A(x,q)) \mid x \in R, q \in Q\},$$

where the functions $\mu_A : R \times Q \to [0,1]$ and $\lambda_A : R \times Q \to [0,1]$ denote the degree of membership and the degree of non membership of each element $(x,q) \in R \times Q$ to the set A, respectively, such that

$$0 \leq \mu_A(x,q) + \lambda_A(x,q)) \leq 1 \text{ for all } x \in R, q \in Q.$$

Definition 2.7. [11] A Q-fuzzy set μ is called a fuzzy R-subnear-ring of R over Q if

(i) $\mu(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$,
(ii) $\mu(xy,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$

for all $x, y \in R$ and $q \in Q$.

Definition 2.8. [11] A Q-fuzzy set μ is called a fuzzy R-subgroup of R over Q if

(i) $\mu(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$,
(ii) $\mu(rx,q) \geq \mu_A(x,q)$,
(iii) $\mu(xr,q) \geq \mu_A(x,q)$

for all $x, y, r \in R$ and $q \in Q$.

Definition 2.9. [11] A Q-fuzzy set μ is called a fuzzy ideal of R over Q if

(i) $\mu(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$,
(ii) $\mu(rx,q) \geq \mu_A(x,q)$,
(iii) $\mu((x+i)y - xy) \geq \mu_A(i,q)$

for all $x, y, i \in R$ and $q \in Q$.

3. Intuitionistic Q-fuzzy ideals of near-rings

Now we introduce the intuitionistic Q-fuzzy ideals of near-rings as follows.

Definition 3.1. An IQFS $A = (\mu_A, \lambda_A)$ of a near-ring R is called an intuitionistic Q-fuzzy subnear-ring of R if

(i) $\mu_A(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(x-y,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q)$,
(ii) $\mu_A(xy,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(xy,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q)$

for all $x, y \in R$ and $q \in Q$.

Definition 3.2. An IQFS $A = (\mu_A, \lambda_A)$ in a near-ring R is called an intuitionistic Q-fuzzy ideal of R if

(i) $\mu_A(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(x-y,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q)$,
(ii) $\mu_A(y+x-y,q) = \mu_A(x,q)$ and $\lambda_A(y+x-y,q) = \lambda_A(x,q)$,
(iii) $\mu_A(rx,q) \geq \mu_A(x,q)$ and $\lambda_A(rx,q) \leq \lambda_A(x,q)$,
(iv) $\mu_A((x+i)y-xy,q) \geq \mu_A(i,q)$ and $\lambda_A((x+i)y-xy,q)) \leq \lambda_A(i,q)$

for all $x, y, i \in R$ and $q \in Q$.

If $A = (\mu_A, \lambda_A)$ satisfies (i), (ii) and (iii) then A is called an intuitionistic Q-fuzzy left ideal of R and if $A = (\mu_A, \lambda_A)$ satisfies (i), (ii) and (iv) then A is called an intuitionistic Q-fuzzy right ideal of R.

Example 3.3. Let $R = \{a, b, c, d\}$ be a non-empty set with two binary operations "$+$" and "$\cdot$" defined as follows:

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(R, +, \cdot)$ is a near-ring.

Define an intuitionistic Q-fuzzy set, $A = (\mu_A, \lambda_A)$ in R as follows

$$\mu_A(a,q) = 1,\ \mu_A(b,q) = 1/3,\ \mu_A(c,q) = 0,\ \mu_A(d,q) = 0,$$

$$\lambda_A(a,q) = 0,\ \lambda_A(b,q) = 1/3,\ \lambda_A(c,q) = 1,\ \lambda_A(d,q) = 1 \quad \text{for all } q \in Q.$$

Then clearly $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy ideal of a near-ring R.

Basic properties of an intuitionistic Q-fuzzy ideal of R are proved in the following theorem.

Theorem 3.4. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy ideal of R then*

(i) $\mu_A(0,q) \geq \mu_A(x,q)$ *and* $\lambda_A(0,q) \leq \lambda_A(x,q)$,
(ii) $\mu_A(-x,q) = \mu_A(x,q)$ *and* $\lambda_A(-x,q) = \lambda_A(x,q)$,
(iii) $\mu_A(x-y,q) \geq \mu_A(0,q) \Rightarrow \mu_A(x,q) = \mu_A(y,q)$,
(iv) $\lambda_A(x-y,q) \leq \lambda_A(0,q) \Rightarrow \lambda_A(x,q) = \lambda_A(y,q)$

for all $x, y \in R$ and $q \in Q$.

Proof. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic Q-fuzzy ideal of R.

(i) $\mu_A(0, q) = \mu_A(x - x, q) \geq \mu_A(x, q) \wedge \mu_A(x, q)$.

Therefore $\mu_A(0, q) \geq \mu_A(x, q)$ for all $x \in R$ and $q \in Q$.

Similarly $\lambda_A(0, q) \leq \lambda_A(x, q)$ for all $x \in R$ and $q \in Q$.

(ii) $\mu_A(-x, q) = \mu_A(0 - x, q) \geq \mu_A(0, q) \wedge \mu_A(x, q) = \mu_A(x, q)$.

Now $\mu_A(x, q) = \mu_A(-(-x), q) \geq \mu_A(-x, q) \geq \mu_A(x, q)$. Thus we get $\mu_A(-x, q) = \mu_A(x, q)$ for all $x \in R, q \in Q$.

Similarly $\lambda_A(-x, q) = \lambda_A(x, q)$ for all $x \in R, q \in Q$.

(iii) We have $\mu_A(x - y, q) \geq \mu_A(0, q)$. But $\mu_A(0, q) \geq \mu_A(x - y, q)$. Thus $\mu_A(x - y, q) = \mu_A(0, q)$.

Now consider

$$\begin{aligned}
\mu_A(x, q) &= \mu_A(x - y + y, q) \\
&= \mu_A((x - y) + y, q) \\
&\geq \mu_A(x - y, q) \wedge \mu_A(y, q) \\
&= \mu_A(0, q) \wedge \mu_A(y, q) \\
&= \mu_A(y, q).
\end{aligned}$$

Similarly we can prove that $\mu_A(y, q) \geq \mu_A(x, q)$. Hence $\mu_A(x, q) = \mu_A(y, q)$ for all $x, y \in R, q \in Q$. ■

A necessary and sufficient condition for $\chi = (\chi_I, \chi_I^c)$ to be an intuitionistic Q-fuzzy ideal of R is given in the following theorem.

Theorem 3.5. *Let R be a near-ring and χ_I be a characteristic function of a subset I of R. Then $\chi = (\chi_I, \chi_I^c)$ is an intuitionistic Q-fuzzy ideal of R if and only if I is an ideal of R.*

Proof. Let I be a right ideal of R.

(1) Let $q \in Q$ and $x, y \in R$. Then we have the following three cases.

Case 1. Let $x, y \in I$. Then $\chi_I(x, q) = 1, \chi_I(y, q) = 1, \chi_I(x - y, q) = 1$ and $\chi_I^c(x, q) = 0, \chi_I^c(y, q) = 0, \chi_I^c(x - y, q) = 0$. Therefore $\chi_I(x - y, q) \geq \chi_I(x, q) \wedge \chi_I(y, q)$ and $\chi_I^c(x - y, q) \leq \chi_I^c(x, q) \vee \chi_I^c(y, q)$ for all $x, y \in R$ and $q \in Q$.

Similarly we can easily obtain the following two cases

Case 2. Let $x \in I$ and $y \notin I$.

Case 3. Let $x, y \notin I$.

Thus in any case we have $\chi_I(x - y, q) \geq \chi_I(x, q) \wedge \chi_I(y, q)$ and $\chi_I^c(x - y, q) \leq \chi_I^c(x, q) \vee \chi_I^c(y, q)$ for all $x, y \in R$ and $q \in Q$.

(2) *Case 1.* Let $y \in R$ and $x \in I$.

Since $y + x - y \in I$ for all $y \in R$ and $x \in I$. Therefore $\chi_I(y + x - y) = 1 = \chi_I(x)$, i.e. $\chi_I(y + x - y, q) = 1 = \chi_I(x, q)$ and $\chi_I^c(y + x - y, q) = 0 = \chi_I^c(x, q)$.

Case 2. Let $y \in R$ and $x \notin I$. Then $\chi_I(y+x-y,q) = 0 = \chi_I(x,q)$ and $\chi_I^c(y+x-y,q) = 1 = \chi_I^c(x,q)$. Hence in any case we get $\chi_I(y+x-y,q) = \chi_I(x,q)$ and $\chi_I^c(y+x-y,q) = \chi_I^c(x,q)$ for all $x, y \in R$ and $q \in Q$.

(3) Let $x, y \in R$.

Case 1. Let $i \in I$.

Since I is a right ideal of $R, (x+i)y - xy \in I$ for all $x, y \in R$ and $i \in I$. Therefore $\chi_I((x+i)y - xy, q) = 1, \chi_I(i,q) = 1$ and $\chi_I^c((x+i)y - xy, q) = 0, \chi_I^c(i,q) = 0$. Thus $\chi_I((x+i)y - xy, q) \geq \chi_I(i,q)$ and $\chi_I^c((x+i)y - xy, q) \leq \chi_I^c(i,q)$ for all $x, y, i \in R, q \in Q$.

Case 2. Let $i \notin I$. Then $\chi_I((x+i)y - xy, q) = \chi_I(i,q) = 0$ and $\chi_I^c((x+i)y - xy, q) = \chi_I^c(i,q) = 1$.

Hence $\chi_I((x+i)y - xy, q) \geq \chi_I(i,q)$ and $\chi_I^c((x+i)y - xy, q) \leq \chi_I^c(i,q)$ for all $x, y, i \in R, q \in Q$. Thus $\chi = (\chi_I, \chi_I^c)$ is an intuitionistic Q-fuzzy right ideal of R.

Similarly if I is a left ideal of R then we can easily prove that $\chi = (\chi_I, \chi_I^c)$ is an intuitionistic Q-fuzzy left ideal of R.

Conversely let $\chi = (\chi_I, \chi_I^c)$ be an intuitionistic Q-fuzzy ideal in R. Then we shall prove that I is an ideal of R.

(i) Since $\chi = (\chi_I, \chi_I^c)$ is an intuitionistic Q-fuzzy ideal of R. Therefore $\chi_I(x - y, q) \geq \chi_I(x,q) \wedge \chi_I(y,q)$ and $\chi_I^c(x-y,q) \leq \chi_I^c(x,q) \vee \chi_I^c(y,q)$ for all $x, y \in R$ and $q \in Q$.

If $x, y \in I$ then $\chi_I(x,q) \wedge \chi_I(y,q) = 1$ implies that $\chi_I(x-y,q) = 1$. Therefore $x, y \in I \Rightarrow x - y \in I$. Hence $(I, +)$ is a subgroup of $(R, +)$.

(ii) Since $\chi_I(y+x-y,q) = \chi_I(x,q)$ and $\chi_I^c(y+x-y,q) = \chi_I^c(x,q)$, therefore clearly if $x \in I$ and $y \in R$ then $\chi_I(y+x-y,q) = \chi_I(x,q) = 1$ and $\chi_I^c(y+x-y,q) = \chi_I^c(x,q) = 0$ implies that $y + x - y \in I$ for all $x \in I$ and $y \in R$.

Hence $(I, +)$ is a normal subgroup of $(R, +)$.

(iii) Since $\chi_I(rx,q) \geq \chi_I(x,q)$ and $\chi_I^c(rx,q) \leq \chi_I^c(x,q)$, therefore if $x \in I$ then $\chi_I(rx,q) \geq \chi_I(x,q) = 1 \Rightarrow \chi_I(rx,q) = 1 \Rightarrow rx \in I$ for all $r \in R, x \in I$.

Similarly if $x \notin I$ then $rx \notin I$.

(iv) Since $\chi_I((x+i)y - xy, q) \geq \chi_I(i,q)$ and $\chi_I^c((x+i)y - xy, q) \leq \chi_I^c(i,q)$, therefore if $x \in I$ then

$$\begin{aligned}
&\chi_I((x+i)y - xy, q) \geq \chi_I(i,q) = 1 \\
\Rightarrow\ &\chi_I((x+i)y - xy, q) = 1 \\
\Rightarrow\ &(x+i)y - xy \in I \text{ for all } x, y \in R \text{ and } i \in I.
\end{aligned}$$

Similarly we can prove that if $i \notin I$ then $(x+i)y - xy \notin I$ for all $x, y \in I$.

Hence I is an ideal of R. ∎

For intersection of any number of intuitionistic $Q-$fuzzy ideals of R, we have

Theorem 3.6. *If $\{A_i|i \in I\}$ is a family of intuitionistic Q-fuzzy ideals of R then $\cap\{A_i|i \in I\}$ is an intuitionistic Q-fuzzy ideal of R where I is an index set.*

Proof. Let $x, y, a, r \in R$ and $q \in Q$. Then

$$
\begin{aligned}
(1)\quad (\cap\mu_{Ai})(x-y,q) &= \wedge\{\mu_{Ai}(x-y,q) \mid i \in I\}\\
&\geq (\wedge\{\mu_{Ai}(x,q) \mid i \in I\}) \wedge (\wedge\{\mu_{Ai}(y,q) \mid i \in I\})\\
&= (\cap\{\mu_{Ai}(x,q) \mid i \in I\}) \wedge (\cap\{\mu_{Ai}(y,q) \mid i \in I\}),\\
(\cup\lambda_{Ai})(x-y,q) &= \vee\{\lambda_{Ai}(x-y,q) \mid i \in I\}\\
&\leq (\vee\{\lambda_{Ai}(x,q) \mid i \in I\}) \vee (\vee\{\lambda_{Ai}(y,q) \mid i \in I\})\\
&= (\cup\{\lambda_{Ai}(x,q) \mid i \in I\}) \vee (\cup\{\lambda_{Ai}(y,q) \mid i \in I\}),
\end{aligned}
$$

$$
\begin{aligned}
(2)\ (\cap\mu_{Ai})(y+x-y,q) &= \wedge\{\mu_{Ai}(y+x-y,q) \mid i \in I\}\\
&= \wedge\{\mu_{Ai}(x,q) \mid i \in I\}\\
&= \cap\{\mu_{Ai}(x,q) \mid i \in I\},\\
(\cup\lambda_{Ai})(y+x-y,q) &= \vee\{\lambda_{Ai}(y+x-y,q) \mid i \in I\}\\
&= \vee\{\lambda_{Ai}(x,q) \mid i \in I\}\\
&= \cup\{\lambda_{Ai}(x,q) \mid i \in I\},
\end{aligned}
$$

$$
\begin{aligned}
(3)\quad (\cap\mu_{Ai})(rx,q) &= \wedge\{\mu_{Ai}(rx,q) \mid i \in I\}\\
&\geq \wedge\{\mu_{Ai}(x,q) \mid i \in I\}\\
&= \cap\{\mu_{Ai}(x,q) \mid i \in I\},\\
(\cup\lambda_{Ai})(rx,q) &= \vee\{\lambda_{Ai}(rx,q) \mid i \in I\}\\
&\leq \vee\{\lambda_{Ai}(x,q) \mid i \in I\}\\
&= \cup\{\lambda_{Ai}(x,q) \mid i \in I\},
\end{aligned}
$$

$$
\begin{aligned}
(4)\quad (\cap\mu_{Ai})((x+a)y-xy,q) &= \wedge\{\mu_{Ai}((x+a)y-xy,q) \mid i \in I\}\\
&\geq \wedge\{\mu_{Ai}(x,q) \mid i \in I\}\\
&= \cap\{\mu_{Ai}(x,q) \mid i \in I\},\\
(\cup\lambda_{Ai})((x+a)y-xy,q) &= \vee\{\lambda_{Ai}((x+a)y-xy,q) \mid i \in I\}\\
&\leq \vee\{\lambda_{Ai}(x,q) \mid i \in I\}\\
&= \cup\{\lambda_{Ai}(x,q) \mid i \in I\}
\end{aligned}
$$

for all $x, y, a, r \in R$ and $q \in Q$.
Hence $\cap\{A_i \mid i \in I\}$ is an intuitionistic Q-fuzzy ideal of R. ■

An intuitionistic Q-fuzzy ideal A of R induces another intuitionistic Q-fuzzy ideal of R is shown in the following theorem.

Theorem 3.7. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy ideal of R then*

(i) *$oA = (\mu_A, \mu_A^c)$ is also an intuitionistic Q-fuzzy ideal of R;*
(ii) *$\bullet A = (\lambda_A^c, \lambda_A)$ is also an intuitionistic Q-fuzzy ideal in R.*

Proof. Let $x, y, a, r \in R$ and $q \in Q$.

(i) (1) $\mu_A^c(x-y,q) = 1-\mu_A(x-y,q) \le 1-(\mu_A(x,q)\wedge \mu_A(y,q))$.
$= (1-\mu_A(x,q))\vee(1-\mu_A(y,q)) = \mu_A^c(x,q)\vee \mu_A^c(y,q)$.
(2) $\mu_A^c(y+x-y,q) = 1-\mu_A(y+x-y,q) = 1-\mu_A(x,q) = \mu_A^c(x,q)$.
(3) $\mu_A^c(rx,q) = 1-\mu_A(rx,q) \le 1-\mu_A(x,q) = \mu_A^c(x,q)$.
(4) $\mu_A^c((x+a)y-xy,q) = 1-\mu_A((x+a)y-xy,q) \le 1-\mu_A(a,q) = \mu_A^c(a,q)$.
(ii) (1) $\lambda_A^c(x-y,q) = 1-\lambda_A(x-y,q) \ge 1-(\lambda_A(x,q)\vee \lambda_A(y,q))$.
$= (1-\lambda_A(x,q))\wedge(1-\lambda_A(y,q)) = \lambda_A^c(x,q)\wedge \lambda_A^c(y,q)$.
(2) $\lambda_A^c(y+x-y,q) = 1-\lambda_A(y+x-y,q) = 1-\lambda_A(x,q) = \lambda_A^c(x,q)$.
(3) $\lambda_A^c(rx,q) = 1-\lambda_A(rx,q) \ge 1-\lambda_A(x,q) = \lambda_A^c(x,q)$.
(4) $\lambda_A^c((x+a)y-xy,q) = 1-\lambda_A^c((x+a)y-xy,q) \ge 1-\lambda_A(a,q) = \lambda_A^c(a,q)$.
Hence $oA = (\mu_A,\mu_A^c)$ and $\bullet A = (\lambda_A^c,\lambda_A)$ are intuitionistic Q-fuzzy ideals of R. ■

From the above theorem we obtain the following

Theorem 3.8. *$A = (\mu_A,\lambda_A)$ is an intuitionistic Q-fuzzy ideal of R if and only if oA and $\bullet A$ are intuitionistic Q-fuzzy ideals of R.*

For a given $A = (\mu_A,\lambda_A)$, we define

$R\mu_A = \{x\in R \mid \mu_A(x,q) = \mu_A(0,q)\}$ and $R\lambda_A = \{x\in R \mid \lambda_A(x,q) = \lambda_A(0,q)\}$.

We prove an important property of $R\mu_A$ and $R\lambda_A$ in the following theorem.

Theorem 3.9. *If an IQFS $A = (\mu_A,\lambda_A)$ is an intuitionistic Q-fuzzy ideal of R then the sets $R\mu_A$ and $R\lambda_A$ are ideals of R for all $q\in Q$.*

Proof. Let $A = (\mu_A,\lambda_A)$ be an intuitionistic Q-fuzzy ideal of R. Let $x,y\in R\mu_A$ and $q\in Q$. Then $\mu_A(x,q) = \mu_A(0,q), \mu_A(y,q) = \mu_A(0,q)$.
(i) Now $0\in R$ therefore $0\in R\mu_A$. Hence $R\mu_A$ is a non-empty subset of R.
(ii) Since $A = (\mu_A,\lambda_A)$ is an intuitionistic Q-fuzzy ideal of R,

$$\mu_A(x-y,q) \ge \mu_A(x,q)\wedge \mu_A(y,q) = \mu_A(0,q).$$

But $\mu_A(0,q)\ge \mu_A(x-y,q)$. Therefore $\mu_A(x-y,q) = \mu_A(0,q)$. Thus $x-y\in R\mu_A$ for all $x,y\in R$. Hence $(R\mu_A,+)$ is a subgroup of $(R,+)$.
(iii) Again since $A = (\mu_A,\lambda_A)$ is an intuitionistic Q-fuzzy ideal of R,

$$\mu_A(y+x-y,q) = \mu_A(x,q) = \mu_A(0,q).$$

Therefore $y+x-y\in R\mu_A$ for all $x,y\in R$. Hence $(R\mu_A,+)$ is a normal subgroup of $(R,+)$.
(iv) Since $\mu_A(rx,q)\ge \mu_A(x,q)$ for all $x,r\in R$ and $q\in Q$, therefore $\mu_A(rx,q)\ge \mu_A(x,q)$. But $\mu_A(0,q)\ge \mu_A(rx,q)$. Hence $\mu_A(rx,q) = \mu_A(0,q)$ implies that $rx\in R\mu_A$ for all $r\in R, x\in R\mu_A$, i.e., $RR\mu_A\subseteq R\mu_A$.

Hence $R\mu_A$ is a left ideal of R.

(v) Next $\mu_A((u+i)v-uv,q) \geq \mu_A(i,q) = \mu_A(0,q)$ for all $u,v \in R, i \in R\mu_A$. But $\mu_A(0,q) \geq \mu_A((u+i)v-uv,q)$. Therefore $\mu_A((u+i)v-uv,q) = \mu_A(0,q)$ for all $u,v \in R, i \in R\mu_A$. Hence $(u+i)v-uv \in R\mu_A$ for all $u,v \in R$ and $i \in R\mu_A$. Thus $R\mu_A$ is a right ideal of R. ■

Theorem 3.10. *If an IQFS $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy ideal of R then the sets $U(\mu_A, t)$ and $L(\lambda_A, t)$ are ideals of R for all $q \in Q, t \in I_m(\mu_A) \cap I_m(\lambda_A)$.*

Proof. Let $q \in Q$ and $t \in I_m(\mu_A) \cap I_m(\lambda_A)$. We have

$$U(\mu,t) = \{x \in R \mid \mu_A(x,q) \geq t, q \in Q\},$$
$$L(\mu,t) = \{x \in R \mid \lambda_A(x,q) \leq t, q \in Q\}.$$

(i) Let $x,y \in U(\mu,t)$. Then $\mu_A(x,q) \geq t$ and $\mu_A(y,q) \geq t$. Since $\mu_A(x-y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q) \geq t$, therefore $x-y \in U(\mu,t)$ for all $x,y \in U(\mu,t)$. Hence $(U(\mu,t),+)$ is a subgroup of $(R,+)$.

(ii) Let $x \in U(\mu,t)$ and $y \in R$. Again since $\mu_A(y+x-y,q) = \mu_A(x,q) \geq t$, therefore $y+x-y \in U(\mu,t)$ for all $x \in U(\mu,t)$ and $y \in R$. Hence $(U(\mu,t),+)$ is a normal subgroup of $(R,+)$.

(iii) Let $x \in U(\mu,t)$ and $r \in R$. Then $\mu_A(rx,q) \geq \mu_A(x,q) \geq t$ implies $rx \in U(\mu,t)$ for all $x \in U(\mu,t)$ and $r \in R$, i.e., $RU(\mu,t) \subseteq U(\mu,t)$. Hence $U(\mu,t)$ is a left ideal of R.

(iv) Let $i \in U(\mu,t)$ and $x,y \in R$. Then $\mu_A((x+i)y-xy,q) \geq \mu_A(i,q) \geq t$. Therefore $(x+i)y-xy \in U(\mu,t)$ for all $i \in U(\mu,t)$ and $x,y \in R$. Hence $U(\mu,t)$ is a right ideal of R. ■

As in Theorem 3.10 we have

Theorem 3.11. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy subset of R such that all non-empty level sets $U(\mu_A,t)$ and $L(\lambda_A,t)$ are ideals of R then $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy ideal of R.*

Proof. Suppose $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy set of R such that all non-empty level sets $U(\mu_A,t)$ and $L(\lambda_A,t)$ are ideals of R.

(i) Let $t_0 = \mu_A(x,q) \wedge \mu_A(y,q)$ and $t_1 = \lambda_A(x,q) \vee \lambda_A(y,q)$ for $x,y \in R, q \in Q$. Then $x,y \in U(\mu_A,t_0)$ also $x,y \in L(\mu_A,t_1)$. Therefore $x-y \in U(\mu_A,t_0)$ and $x-y \in L(\mu_A,t_1)$. Thus $\mu_A(x-y,q) \geq t_0 = \mu_A(x,q) \wedge \mu_A(y,q)$ and $\mu_A(x-y,q) \leq t_1 = \mu_A(x,q) \vee \mu_A(y,q)$.

(ii) Let $t_0 = \mu_A(x,q)$ and $t_1 = \lambda_A(x,q)$. Then $x \in U(\mu_A,t_0)$ also $x \in L(\lambda_A,t_1)$ and $y \in R$.

Since $U(\mu_A,t)$ and $L(\lambda_A,t)$ are ideals of R, they are normal. Therefore $y+x-y \in U(\mu_A,t_0)$ and $y+x-y \in L(\lambda_A,t_1)$ for all $x \in U(\mu_A,t_0)$.

Now $x \in L(\lambda_A,t_1)$ and $y \in R$

$$\Rightarrow \mu_A(y+x-y,q) \geq \mu_A(x,q) \text{ and } \lambda_A(y+x-y,q) \leq \lambda_A(x,q).$$

(iii) Let $t_2 = \mu_A(x,q)$ and $t_3 = \lambda_A(x,q)$ for $x \in R$ and $q \in Q$. Then $x \in U(\mu_A, t_2)$ and $x \in L(\lambda_A, t_3)$.

Since $U(\mu_A, t_2)$ and $L(\lambda_A, t_3)$ are left ideals of $R, rx \in U(\mu_A, t_2)$ and $rx \in L(\lambda_A, t_3)$ for $r \in R$. Hence $\mu_A(rx,q) \geq t_2 = \mu_A(x,q)$ and $\lambda_A(rx,q) \leq t_3 = \lambda_A(x,q)$.

(iv) Let $t_2 = \mu_A(i,q)$ and $t_3 = \lambda_A(i,q)$ for $x \in R$ and $q \in Q$. Then $i \in U(\mu_A, t_2)$ and $i \in L(\lambda_A, t_3)$.

Since $U(\mu_A, t)$ is a right ideal of R,

$$(x+i)y - xy \in U(\mu_A, t_2) \text{ for all } x, y \in R \text{ and } i \in U(\mu_A, t_2)$$

and

$$(x+i)y - xy \in L(\lambda_A, t_3) \text{ for all } x, y \in R \text{ and } i \in L(\lambda_A, t_3).$$

Therefore $\mu_A((x+i)y - xy, q) \geq t_2 = \mu_A(i,q)$ and

$$\lambda_A((x+i)y - xy, q) \leq t_3 = \lambda_A(i,q) \text{ for all } x, y, i \in R \text{ and } q \in Q.$$

Hence $A = (\mu_A, \lambda_A)$ is an intuitionistic Q-fuzzy ideal of R. ■

4. Conclusion

Near-ring theory has many applications in the study of permutation groups, block schemes and projective geometry. Near-rings provide a non-linear analogue to the development of Linear Algebra, combinatorial problems and useful for agricultural experiments. In this paper we have presented the notion of intuitionistic Q-fuzzy ideals of near-rings and derived the properties of these ideals.

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